# The Logic of Aristotle

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**Abstract**

Twenty three centuries ago Aristotle made a comprehensive survey of the philosophical literature of his time, which included all the scientific literature. From this he distilled a series of texts now called the *Organon*. These texts are amongst the earliest of any we know that deal with what is now called logic: the formal characterization of what constitutes a valid argument.

The main use to which Aristotle put logic was not the establishment of indubitable truth, which he considered to be impossible. Rather it was to find the most natural axiomatizations which yield the intuitive understanding of the evedent truths of a subject. Aristotle considered mathematics to be an empirical science like any other, except that it deals with abstract ideas which are exact. The same principle of a natural axiomatization is applied to the other sciences.

Using logic to reason about scientific reasoning in a completely general way, Aristotle was able to explicitly describe a general method for discovering the natural axiomatization of any particular subject. He was thus able to explain the necesity of certain general features of scientific knowledge as a whole, the most important of which is that there can be no single fundamental science which explains all other sciences. The reason is formal: because every science is based on demonstrative knowledge, and because all knowledge depends on other knowledge, not all knowledge can be demonstrated. Some self-evident facts in any science must be assumed to avoid either an infinite regress or circular reasoning. There can be no single science of everything simply because the self-evident truths of one science are not those of another.

The necessity of assuming some basic truths appropriate to the generic subject of the science means that each science can only demonstrate knowledge in that one subject, and only in others in so far as the they share a common subset of basic truths. The evident fact of there being more than one generic subject means that there cannot be just one science. For example, take physics and arithmetic. These cannot share the same set of self-evident truths because arithmetic is the science of collections of things considered as numbers, and physics is the science of things considered as substance. The self-evident truths concerning collections of things considered as numbers are different from those of things considered as substances because elements of collections of actual substance may join and divide so they do not necessarily have arithmetical properties.

Aristotle held that it was the task of philosophy to make clear the relation between the other sciences and to demonstrate the common notions which are assumed by many sciences, e.g. that two things equal to the same thing are equal to each other, etc. But philosophy is not a single science of everything because it does not assume the basic truths of the other sciences. Rather philosophy assumes the basic truths appropriate to itself and also the demonstrated knowledge obtained in the other sciences, but not their basic truths. Philosophy is the

science of being as pure being whereas the other sciences are sciences of things considered as being in some particular sense: for example botany is the science of being in the sense of being a plant, and physiology is the science of being in the sense of being an animal. Medicine is the science of being in the sense of being healthy, geometry is the science of being as being in some definite place. Physics is the science of being as being a substantial object. All of these sciences have essentially different self-evident truths. All sciences for example, assume the existence of their objects of discourse, they don't set out to explain how these objects come to be because that is prior to the science itself: biology cannot explain the origin of life because it would have to do so in terms of something prior to life, and such a thing would not be proper to the subject of biology.

The various sciences are therefore each natural philosophies, where knowledge of some particular mode of being is pursued for its own sake. Philosophy is the science where knowledge of being as pure being is pursued for its own sake. Philosophy cannot be done apart from the other sciences because all the particular knowledge about being belongs to them. So the elements of philosophical knowledge are the particular types of knowledge of the different sciences. The only kind of philosophy is therefore the philosophy of science and philosophy is an empirical science like any other, just more abstract. But ultimately it must rest on actual experience.

I argue that Aristotle was essentially correct and that the only reason this is not something that is more widely understood is that he has been misinterpreted by those who failed to understand a fundamental tenet of his philosophy, one he shared with Plato, which is that true knowledge cannot be represented completely by any collection of opinions or beliefs. In a nutshell, the argument goes like this: Philosophy is love of wisdom, which is true knowledge. True love of some whole thing is love of all of its constituent parts indiscriminately. The pursuit of knowledge to any particular end will necessarily discriminate between that part of knowledge relevant to the particular end and other parts not apparently relevant. Thus the true knowledge pursued by the philosopher cannot be to any end except itself, and therefore philosophical knowledge is knowledge about all things. Demonstration relies on self-evident basic truths which cannot themselves be demonstrated. The self-evident truths are only self-evident in respect of the particular sense in which they are true, which is according to the particular subject genus. Thus demonstration can only be in respect of some part of knowledge, not the whole, and the knowledge that the philosopher seeks cannot be demonstrated or proved. Moreover, since the particular sciences in turn use the common notions which are demonstrated propositions of philosophy, a similar argument holds for the other sciences individually. Because they take as basic truths the common notions of philosophy they cannot individually demonstrate their particlar part of knowledge as a whole. Therefore true knowledge as a whole can ever be the subject of demonstration or proof of any kind. Thus no actual collection of beliefs or opinions could constitute true knowledge, however certain may be the proponents of those beliefs or however soundly they may have been demonstrated.

Aristotle's logic is necessary in order to understand the logical structure of science because it makes a crucial distinction between the symbolic representation of a fact and the fact itself, which is something that a purely symbolic representation of logic cannot do. When we make this distinction we see that Aristotle's philosophy is able to explain some of the most deeply puzzling problems in the foundations of modern science, and that his method is in fact a sound basis on which resolve the problems and make forward progress.

# 1 Introduction

In logic and mathematics many have a strong sense that, although abstract, there is nevertheless a kind of reality behind the objects of their study. Although there are many different possible symbolic representations, and though they may be shown to be essentially equivalent, the feeling is that there is one way of presenting the abstraction which is the true way. Exactly which presentation this is depends not so much on the subject itself as on its relations to other subjects. It is found for example that certain axiomatisations are more natural than others in that they lead to more intuitive connections with related subjects. The sign of true mathematical talent is the ability to find the natural formulation of a problem: that which yields the intuitive reasons why the theorems hold. The judgements about what is the better formulation of a problem are often in aesthetic terms like beauty and elegance.

In the hard sciences, the sense is that the objects of study have a more concrete physical reality. The physicist has the sense that this concrete foundation is the medium of causality: 'What happens is what happens because these *things* actually bump into each other.' The physical reality is thus thought to be a consequence of the conjunction of *material cause*, which are the things with their inherent attributes of mass, charge, color etc., and the *efficient cause* which are the initial conditions or states of those things: their positions and states of motion, perhaps. The reality of the mathematician is formal: 'What happens is what happens because these *forms* are those of logical consequence.' in other words because it could not possibly be otherwise. Thus the physicist is also happy to accept the mathematical explanation for the forms of the things he studies. In doing this the physicist is admitting the notion of a *formal cause* as well as the material and efficient causes. An example is the Special Theory of Relativity which takes the assumptions of causality and relativity of inertial frames[1] to conclude that simultanaeity is relative to the observer's state of inertial motion. Now the physicist finds that he can explain even more if he admits *final cause*. For example, if he includes in his theory his intention to detect the presence or not of quantum particles in a certain place and during some time interval then it is able to describe almost all phenomena that occur in his laboratory.

However at each stage he loses one feature of his original explanatory basis. Upon including formal cause he loses the sense in which the efficient cause is absolute, because the order of events is no longer determined globally. On including final cause he loses the sense in which material cause is absolute, because the material cause must propogate backwards with respect to the efficient cause. As a result the phenomena that are described are essentially non-deterministic; and this is just a logical consequence of having admitted these two extra types of causality.

Aristotle's philosophy of science is based on these four senses of causality and his logic is the formal framework in which he describes the scientific method which has the express purpose of discovering the natural axiomatisations of any empirical science, including all of mathematics. On this basis I claim that a careful study of Aristotle's logic and philosophy of science will help to resolve some of the most difficult

[1]The idea of causality in this sense is that one event is the cause of another distinct event and it is not hard to see that there must be a maximum velocity of propagation of effects otherwise all events would occur at once. Whence the joke about time being 'what stops everything from happening at once'. If there is a maximum speed then the princile of relativity says that in addition it must be an absolute speed because otherwise the maximum could always be exceeded with respect to some suitably chosen moving reference frame. The speed of light $c$ appears as an absolute constant in Maxwell's equations describing propogation of electromagnetic energy in space, so this was taken by Einstein to be the maximum velocity of the propagation of any energetic effect through a vacuum.

foundational problems in both mathematics and the natural sciences. The substance of my claim is that Aristotle used formal logic to analyse the axiomatic structure of the empirical sciences and that the results show that certain features of the formal representation of scientific theories are necessary in order for there to be any possibility of a consistent interpretation. These necessary features are absent in all recent attempts to explain the foundations of mathematics and quantum mechanics.

The reason we need to go all the way back to Aristotle is that from around 350 BCE, when Aristotle wrote, until around the middle of the nineteenth century, logic remained essentially unchanged. Then Boole introduced mathematical logic in an attempt to treat logic as a branch of applied mathematics using algebraic equations to capture logical deduction and submit it to a mechanical calculus. It was argued by Boole and then others that mathematical logic was a generally much more expressive and more comprehensive system than that of the traditional logic. Philosophical logicians[2] raised serious doubts that this was in fact so, but they were largely ignored and so mathematical logic very quickly became the theatre for a series of startling and exciting developments. Philosophical logic was thereafter almost completely ignored.

The problem with mathematical logic is that it fails to capture the distinction between the symbolic representation and the fact.[3] This is just because it is itself merely a symbolic representation. This subtle discrimination is vital to Aristotle's use of logic in the philosophy of science because we need to reason about potential change within the scientific theory itself. The language of predicates and terms which we would call the 'non-logical symbols' are not fixed in Aristotelian logic. This is because the logic is intended to be used to describe the process by which we decide how to redefine terms or define new terms. In order to do this it is necessary to be able to reason about the symbolic representation of the fact as well as about the fact itself, and thus to make a distinction between the two.

## 2 Caveat

Aristotle's thought is a league above that of anyone else I know of. It is the failure to appreciate this which leads so many to assume that, because they do not immediately understand what he wrote, it must be wrong. For example Russell wrote that almost everything Aristotle said had to be overturned before any new progress could be made. And more recently Martin-Löf is able to say of what he calls Aristotle's 'definition of truth' that 'A moment's thought shows that this is wrong', and he does not see it necessary to explain why two thousand three hundred years had to pass before anyone noticed so obvious an error.[4] Such opinions are commonly expressed, often by people who have never even attempted read Aristotle themselves, and the more frequently they are expressed, the less inclined anyone is to persist in trying to come to an understanding of Aristotle. And that is pretty much how opinion stands today: Aristotle was wrong about almost everything, they say. The nature of philosophy though is such that whether he was right or not is not actually a matter of opinion, but of fact. So if at all possible, avoid reading commentaries by others because they

[2]Most notably Lewis Carroll in [1], the entire text of which appears in Appendix A.

[3]This was noticed by Russell in [6], but he failed to correctly identify the solution to the problem: he just eliminated all references to the facts!

[4]It is not a coincidence that the definition of truth which Martin-Löf offers eliminates the notion of truth as being anything distinct from proof. Thus he follows Russell in discarding all references to the facts.

are more likely to mislead than not. Go straight to the writing of Aristotle himself and come back to the commentary when you feel you understand the man himself.

This too is a commentary, so stop reading right here, and go and read Aristotle. When you have done this, come back and you will be in a position to judge whether what I claim here is accurate or not.

## 3 The atomic revolution in science

The latter half of the nineteenth century saw a sea-change in scientific reasoning, one in which it finally turned away completely from philosophy and in which empiricism was to be the only basis for knowledge. In science this empirical foundation would be experiment and in mathematics it was taken to be the finitistic formal proof described by Hilbert. This was of the kind that would eventually find its way into today's automated reasoning systems and proof assistants: finite physical realizations of formal systems of mathematical proof.

The atomic hypothesis was clearly an important component of this change. Before this, all natural philosophy had been strictly phenomenological: describing the observed relations between events in the laboratory, but following Newton in saying 'I frame no hypotheses' when pressed to explain the reasons why those particular relations held and not others. But with the advent of Maxwell's spectacularly successful theory it became possible to contemplate that the laws of electrodynamics might be a significant part of the laws that could ultimately explain the phenomenology of chemical reaction in terms of the elements; hitherto known only indirectly via the phenomena of certain quantitative relations which held during chemical reactions. Thus in addition to providing a theory of electrodynamics, Maxwell's theory provided the first symbolic language in which scientific statements about the atomic realm might be framed, and Newtonian mechanics provided a calculus which could be used to explore their implications.

With this possibility it did not seem to be too unreasonable to hope that these ultimate building blocks of physical matter might reveal the complete laws of interaction of matter and energy and thereby explain all phenomena including the fact of human experience itself. These physical laws would then be the ultimate basis not just for scientific truth, but for *all* truth including mathematical truth. Thus Laplace imagined a *daemon* who knew the positions and velocities of all the fundamental particles and was able to use the laws of dynamics to predict and retrodict all events that ever would or ever had ocurred, presumably including even his writing of those very words. The hypothesis was that the physical laws must be deterministic and this is indeed a necessary condition if they are to be considered as an explanation for anything: if the laws are not deterministic then what happens is not ultimately explained by the laws themselves, but either by chance or by some particular circumstance which these laws would not explain.

It was then not considered to be particularly far-fetched that these same laws might explain the origin of matter, energy and space-time itself. Thus we came to the widely espoused view that the elementary laws of physics could potentially explain all phenomena, including even the origin of the physical laws themselves, using only some very general 'anthropic principles' such as that which says the Universe should contain observers capable of learning what those laws actually are. The many apparent philosophical problems with this view did not go entirely unnoticed, but the spectacular successes of the atomic theory suggested to many that Nature herself did not seem to have let any of these philosophical problems get in the way; *cf* Feynman's comment 'Philosophy of science is about as useful to scientists as ornithology is to birds.' It was already obvious that the answers that nature seemed to have in this realm were very different from those we would have come up with ourselves. Thus it was thought that if we only pay careful enough attention to what actually happens in the laboratory then all will eventually become clear and we will then have a theory of physical substance which exmplains all events in the Universe at any scale as being due to the fundamental substances interacting mechanically with formal cause relegated to the absolute minimum number of self-evident cosmological principles.

## 4 The atomic revolution in mathematics

The atomic hypothesis affected the philosophy of mathematics profoundly. Until the middle of the nineteenth century almost all mathematics was considered to be applied mathematics.[5] Mathematics had a foundation in the reality of experience. Mathematicians worried about what was true, not about whether mathematics itself was consistent or not. Curiously, the idea of the material foundation of phyical reality resulted in mathematics losing its connection with reality. Some philosophically-minded mathematicians started to worry then about consistency and sought to base it in truth.

By the end of the eighteenth century the phenomenological and essentially formal description offered by Newtonian mechanics had been given a very elegent axiomatic treatment in the mechanics of Lagrange, but this too was purely formal: it did not offer any hope of a 'causal explanation' of Newtonian mechanics, i.e. one in terms of just efficient and material cause. The variational calculus of Lagrange was based on the infinitesimal calculus of Newton and Leibnitz so was given a formal justification by Cauchy but this was in terms of functional analysis. Thus the question arose as to the reality of the numbers that the calculus was supposed to be about. But Cauchy's axiomatisation of analysis is not 'about' the numbers: the subjects of discourse are real-valued functions with certain properties such as limits, continuity and differentiability. There is no need to think of the reals as actually existing separately from these limits. The language is relational and describes processes repeated indefinitely. There was no hope that any theory expressed in this kind of intensional mathematical language could ever constitute a mathematically extensional explanation, say, as to *why* a massive particle experiencing a constant force would undergo constant acceleration so that its velocity would increase as the square of the distance travelled.[6]

The idea that the underlying space of the real numbers was important was motiviated by the belief that there could never be an explanation of Newtonian machanics in terms of material and efficient causes alone unless there was a medium in which these could be seen to be operating, and that medium was expected to be space-time described as sets of real numbers. The search then became one for the set-theoretical foundation for the functional analysis.

It was known by the Pythagoreans that the ratios of whole numbers were not complete with respect to the lengths that could be produced by geometric construction. Thus if $f$ is the positive square root function, the rationals were known not

[5]This is argued convincingly by Maddy in [3].

[6]The extensional mathematical description corresponds to an intensional physical description and *vice versa*.

to be sufficient to prove that $f(x)$ is continuous at 2. There was evidence that the rationals together with the algebraic numbers might not be sufficent either because of the problem of squaring the circle, which involved a geometric construction of a square having the same area as a given circle. By the time of Euclid, the Greeks had developed a geometrical theory of measure which was capable of dealing consistently with all the incommensurate measures that came up in geometric construction.[7]

Cauchy had dealt with this difficulty by using for his definition of the reals a construction of Dedekind's which identified each real number with a pair of open sets of rationals called a 'cut' which bracket the given number. This requires nothing more than the density property of the rationals: that between any two distinct rationals there lies a third rational. Thus the reals were considered to be just the boundaries between the open sets of reals, so the real numbers that exist are exactly the numbers that can be described using some combinations of theorems from Cauchy's formulation of real analysis. This was considered impredicative: the elements of analysis were determined by the theorems of analysis itself. This, it was argued, was no foundation for the belief in the physical reality of Newtonian mechanics. The mathematical theory of real analysis could not be considered to be a basis for a mathematical explanation of Newtonian mechanics in terms of efficient and material cause unless there were an extensional description of the underlying space of real numbers as a collection of individuals each with an independent existence. What was sought was another model of the reals in which the real numbers have an existence independently of the theory of real analysis.

Thus Cantor found himself analysing the sorts of second-order infinite process which lead to the completion of metric spaces of the kind that Cauchy's functional analysis required.[8] The idea was that whenever Cauchy said something like 'The limit of $f(x)$ as $x$ approaches $x_0$ from the left exists if $\forall \varepsilon > 0.\exists \delta > 0.|f(x_0) - f(x_0 - \delta)| < \varepsilon$', that the numbers the quantifiers ranged over had an independent existence. In the process, Cantor produced an axiomatic set theory whch included infinite sets of ordinal numbers counted by cardinal nubers. Most strikingly, Cantor's cardinals included 'uncountably infinite' numbers in the sense that they were the cardinalities of sets that were strictly greater than that of the set of natural numbers. It was clear that most[9] of the elements of these sets have no finite intensional symbolic description. Thus the problem became how to justify the existence of the completed sets when most of their elements could not be described individually.

This background was the motivation for Hilbert's programme: the construction of an axiomatization of mathematics in terms of self-evident finite objects; the elements of mathematics. Hilbert's expectation seems to have been that these ojects would be the natural numbers. It was already evident that problems from disparate branches of mathematics could be turned into problems in elementary number theory, the theory of arithmetic equations using just the whole numbers with addition, multiplication and exponentiation. For example, the question of finding a complete set of topological invariants of a closed three dimensional simplicial manifold can be

[7] *Elements* Book V.

[8] This model uses equivalence classes of *Cauchy sequences* to represent the real numbers and it requires the underlying metric space to be complete in the sense that every Cauchy sequence converges: i.e. the space contains all limits of Cauchy sequences. A sequence $x_i$ is Cauchy if $\forall \varepsilon > 0.\exists N.\forall m, n > N.|x_m - x_n| < \varepsilon$.

[9] Those with finite descriptions can be put in correspondence with the integers and the rest with the open unit interval which can be put in correspondence with the whole of $\mathbb{R}$.

reformulated in terms of a theory of incidence matrices.[10] Given an enumeration of sets of incidence matrices there is a function $f$ of elementary number theory such that for some fixed $x$, the equation $f(m,n) = x$ has a solution if and only if there exists a homeomorphism between the $m$th and the $n$th sets. Clearly $f$ then represents the complete set of invariants we seek, but it is only useful if, given arbitrary $m$ and $n$ we have a practical method to determine whether they are indeed solutions to $f(m,n) = x$. So we have *reduced* the problem in geometric topology to one in elementary number theory: that of finding a practical method to solve equations of the particular form $f(m,n) = x$.

All this was all already well-known. Hilbert's major contribution was the suggestion that if there were an *effective method* for solving any and every set of diophantine equations, and if there were also an effective method for reducing any sufficiently well-defined problem from any domain of mathematics whatsoever to diophantine equations, then there would be an effective method to solve any and every sufficiently well-defined problem in mathematics. This would give the needed finitistic foundation of mathematics, because any effective method must necessarily be finite both in terms of the length of the statements involved in carrying it out and the number of steps to determine the solution.

Hilbert probably had in mind Cantor's set theory. This was sufficiently well-defined that it should be formulable in elementary number theory. Cantor had run into a problem though, because he could not prove the continuum hypothesis which was the assertion that there was no set of cardinality greater than that of the natural numbers $\mathbb{N}$ but also less than that of the real numbers $\mathbb{R}$. The cardinality of the integers is called $\aleph_0$ and that of the real numbers is $2^{\aleph_0}$. Cantor had proved that most[11] of the elements of $\mathbb{R}$ do not have finite symbolic descriptions in any notation whatsoever.[12] So the continuum hypothesis is the hypothesis that there is no way to describe a proper infinite subset $X$ of $\mathbb{R}$ that is neither of equal cardinality to the whole of $\mathbb{R}$, as the unit interval is, nor of equal cardinality to $\mathbb{N}$, as the set of rationals is.

The central idea of set theory is that we only know sets by their elements and in ZFC we don't even have any elements except the empty set. Using an idea of Russell's[13] we can put sets in correspondence with numbers by putting 0 in correspondence with the empty set, and then taking the numbers to be the boundaries between the sets of numbers downward-closed under the predecessor relation. Then any set in ZFC is just a set of natural numbers. Had we been able to describe a set $X$ of cardinality greater than that of $\mathbb{N}$ and less than that of $\mathbb{R}$ then we would have somehow discriminated between at least two individual elements of $\mathbb{R}$ which do not have finite descriptions, because $X$ must contain at least one such, otherwise it would be countable, and it must not contain at least one such, otherwise it would have the same cardinality as $\mathbb{R}$. Set theory would therefore have been shown to have some means of discriminating between sets strictly beyond that which could be achieved by considering their identity as being the collection of individuals that comprise them. Set

[10] This is the example from Church's [2].

[11] Those that have no finite description can be put in one-one correspondence with the whole of $\mathbb{R}$.

[12] Cantor's theorem shows that the cardinality of $\mathscr{P}(\mathbb{N})$ is strictly greater than that of $\mathbb{N}$ and he also showed that there is a bijection from $\mathscr{P}(\mathbb{N})$ onto $\mathbb{R}$. But one can easily construct a bijection between finite sequences of (indices of) symbols and an infinite subset of $\mathbb{N}$ so there are not enough finite strings of symbols in any finite alphabet to describe all elements of $\mathbb{R}$.

[13] This is not exactly how he thought of it, but it seems to have been his idea to use the ancestral of the successor relation to define the natural numbers. See his excellent [5] which he wrote whilst serving a prison sentence.

theory would not be as it is described in the technical jargon: 'the first order theory of one extensional binary relation and equality'. So it seems that the negation of the continuum hypothesis probably shouldn't be provable in set theory as we know it.

Intuitively the Continuum Hypothesis *is* true. This is because we don't construct the set $\mathscr{P}(\mathbb{N})$ in the proof of Cantor's Theorem, we take it as axiomatic that it exists[14] and we just prove *per impossibile* that there is no injection from it into $\mathbb{N}$. So we have the existence of a set which is mostly elements beyond individual finite description. But we hypothesised them into existence as a whole completed infinity, and we didn't need to describe them individually to do that. But if we could describe some proper part of that set of them then we would again have been able to exceed the discriminatory ability of set theory as a theory of extensional equality. So we should not expect to be able to prove that the Continuum Hypothesis is true by contradiction. We can only do it by explicitly constructing uncountable sets, so proving it directly would involve proving that no smaller uncountably infinite set could be conjured whole in this way from some combination of the axioms. This is in effect what Cohen was able to do his famous proof.[15] The Generalised Continuum Hypothesis is the statement that all uncountablly infinite sets are obtained this way: by using the powerset axiom on an infinite set.

## 5 Quantum mechanics

It is now over a century since Einstein suggested that if electromagnetic radiation were physically quantised then it would explain two otherwise quite different phenomena: that of the observed spectrum of black-body radiation and the frequency-dependency of the current in the photoelectric effect. So the very first step on the road that was to lead to modern quantum mechanics was the hypothesis of an ontological explanation for a phenomenological description. The *onta* that were hypothesised were photons, the so-called *gauge bosons* of the electronic field.

This later terminology comes from the formalisation of the process taking phenomenological descriptions in the form of classical field theories and imposing some very general conditions called *gauge invariance*, which amount to little more than a demand that the laws of physics be covariant with respect to things which 'really shouldn't be important.' A classical field theory is a phenomenological description of relations that hold between objects and which depend on their locations in space. It is phenomenological because it is not something that can be said to exist apart from the objects. The gravitational field in Newtonian physics is an example. It can be described as a scalar potential which is due to the positions and masses of the objects whose interaction is describes, but it cannot be said to be a separate thing acting on those objects because there is no conceivable connection that can be realized which allows the field to affect the object. For example in electrostatics the force acting between two charges separated by a distance $r$ varies as $1/r^2$ so at the surface of any charge the field is infinite and entirely due to the that charge itself. The other charges cannot be said to affect it through any actual, substantive mechanism of which the field could be said to be a medium—e.g. the *luminiferous aether*. Thus the vector field describing the forces that act between particles is descriptive but not proscriptive.

[14]This comes from the conjunction of the axiom of Infinty and the Powerset axiom.

[15]I don't know anything more than that the proof involvs sophisticated model theory, the details of which I have never heard being the subject of casual discussion, even amongst the cogniscenti of formal foundations of mathematics.

In the case of electrodynamics the so-called gauge symmetries are described as a global constant phase angle on the complex valued spinor field which describes electron spin states. This is known not to affect the description of the quantum mechanical system in any observable way, because the probablity distributions are the squared amplitudes and these are not affected by a factor $e^{i\theta}$ of unit amplitude applied uniformly. But if this is a global change under which the laws of physics are invariant, then it cannot matter what is chosen locally either, provided it changes continuously from place to place, and provided we know what transformation to make to the mathematical descriptions of the physics as we move from place to place.

Thus we let the arbitrary phase angle be a function of position, replacing $\theta$ with $\theta(x)$ and this imposes a requirement that there be a certain transformation of the differential operators[16] which makes them position-dependent. The result is an operator called the *covariant derivative*, one component of which is a new scalar field called the *mediator* which expresses the particular phase factors at each point. These are arbitrary of course, and we aren't interested in any particular choice of mediator field. What we do is demand of any field equation determining a mediator that it too be invariant under the same gauge symmetry. The idea is to explore the physical significance of this postulated entity, and the only way it could have physical significance would be if it were connected with the local gauge transofrmation which determined it. This guage invariance is not too difficult to arrange in the case of the electron field and the associated gauge field it determines turns out to describe exactly the observed electromagnetic potential which describes the relativistic interaction forces between electrons.

The next step is *quantization* of these classical field theories and the result is the theory of Quantum Electrodynamics which describes the forces between electrons as due to exchange of photons, which are the so-called *gauge bosons* that result from quantizing the gauge field. It is in this quantization process that the phenomenological description given by the classical field theories is turned into an ontological description in terms of fundamental particles. A similar process gives the whole standard model which describes three of the four fundamental forces. The fourth, gravity, is also described by a gauge theory and was developed using exactly these ideas of covariant laws of physics and localization of symmtery invariance.[17]

So what started out as an unobservable feature of the phenomenological description of electron spin state has turned into a causal explanation in terms of forces for the phenomenologically described electron interaction. And this was just by our demanding that the laws of physics be observer independent, which necessarily entails that the theoretically unobservable be empirically undetectable. As such this was a testable consequence of the mathematical theory of electron spin state. The theory was that the electron state is described by a complex scalar field and that the equations of motion this determined were invariant under a global symmetry described by a constant phase factor. This implied that if indeed the mathematical formalism is correct, then whenever the theory is interpreted (i.e. tested by experiment, which is something that always happens at some particular place) the consequences of the global invariance under phase shift should not be detectable.

However, as has often been noted, the description of physics in terms of quantum mechanics has very little in the way of explanatory power. Feynman also famously

[16]These operators are the ones that appear in the equation describing the field. This equation is called a *lagrangian* and it is a compact description of the equations of motion determined by the field.

[17]The problem is that we cannot find one single gauge symmetry that determines the gravitational force together with the other three fundamental forces.

said ‘If anyone tells you they understand quantum mechanics then all you know is that you have met a liar.’ So paying close attention to the way Nature ‘does the reality thing’ using fundamental particles yields some fundamental particles whose actual behaviour is so mysterious that it is apparently not described by any laws whatsoever. Niels Bohr thus insisted that the quantum description of nature is an esentially phenomenological one and this is the basis for the notorious *Copenhagen interpretation* of quantum mechanics.

These two opinions of Feynman, if he held them at the same time, would each justify the other: if a philosopher of science had come along and explained quantum mechanics, Feynman would have taken that to be a verification of both these opinions: that philosophers of science are useless to science because they don’t know what an explanation is. Feynman’s mathematical intuition was so good that I believe he knew that no one would be able to provide an explanation of quantum mechanics using material and efficient cause alone. For some reason he didn’t see that anything other than these would be an explanation.

But if we look carefully at the line of argument then we see that in fact the point at which a causal explanation emerged was exactly the point at which we supposed that the physical theory invariant under global gauge transformations should be observer independent, and so the phenomenological description should be invariant under local gauge symmetries. It is at this point that the causal explanation appears of the phenomena as being the result of gauge forces. So we can see very clearly that the second-order *reason for the explanation* is precisely the fact that the laws should be independent of any accidental circumstance in which any particular observer should find themselves. The relativity principle that is being applied is that the True laws are those that are not dependent on any particular local gauge. But in fact experimental evidence is only ever of some particular local gauge: the invariance under the global gauge transformations is not something that can ever be experimentally verified. This is something that both Plato and Aristotle would have been delighted to learn.

## 6 The Platonic ideal of Truth

The Platonic definition of knowledge is as something strictly beyond any collection of opinions or beliefs, which latter include all things for which we have evidence of any kind. Plato held that the objects of knowledge exist in a different world from that of the objects of sense-perception. Aristotle’s idea of knowledge is very similar, but he differs from Plato in that he asserts that the objects of our knowledge do not exist apart from the substantive objects of our sense-perception.

The obvious question is then ‘if true knowledge is beyond any particular set of beliefs, and if the latter include all those things which we know to be true through any kind of proof whatsoever, then how is it that we can know it even exists?’ This is not supposed to be obvious. Plato could not explain why it is that some people can see that knowledge is real but others cannot.

Aristotle’s answer was that we cannot demonstrate it, but that we can come to know it directly through intuition. Because of the inherence of the objects of knowledge in the substantive objects of sense-perception, Aristotle held that dialectic, geometry, arithmetic, physics and philosophy are all empirical sciences based on a particular form of dialectic argument called *demonstrative syllogism*, together with some particular basic truths, each of which were the *elements* appropriate to that science. We come to know the basic truths through intuition and we develop knowledge as a consequence of a certain type of deductive reasoning. This is called 'knowledge of the demonstrated fact' and the trick is to get all our knowledge of this form to align with real knowledge. It is knowledge of the demonstrated fact that we prove, and in which we place our belief, but real knowledge remains strictly beyond any that could be characterised by demostrative knowledge, i.e. proof or evidence of any kind.

True knowledge then is the structure which is the limit of the particular opinions or beliefs we have about things of which we believe we have scientific knowledge. In the *Prior Analytics* Aristotle analyses the form of dialectic syllogisms and identifies those forms which are valid and those which are not. In the *Posterior Analytics*,[18] he uses the results of this formal analysis as a basis for elucidating the nature of *demonstrative syllogism*, which is the type of reasoning that produces scientific knowledge. The results of this analysis are formal conditions which constrain the axiomatic structure of the sciences.

The use to which Aristotle puts the formal logic is in deducing the abstract forms of the axiomatic structure of the sciences. The truth or falsity of particular statements is incidental to this: the primary role of logic is in helping us to identify what are the appropriate axioms to use in formulating scientific knowledge so that we can better know when our knowledge is in fact not true knowledge. This is revealed to us through contradictions of the validly-drawn conclusions with what we actually know through sense-perception (i.e. intuition). Such contradictions are to be resolved by choosing different axiomatizations, re-arranging the elements of the science, and perhaps identifying new sciences to accommodate the new structure.

The ultimate basis for all scientific knowledge is not proof, but intuition. Intuition tells us that the elements of a science are basic truths that do not need proof. And ultimately it is intuition by which one comes to know knowledge itself as something real, despite it being strictly beyond proof. Thus the axiomatic structure of geometry, for example, is what the text of Euclid's *Elements* is really concerned with. The indubitable truth of the propositions is a mere side-effect of this. This is why Euclid does not explicitly annotate the deductions with the propositions on which they depend: these annotations were only added by later commentators. The text is not intended to establish the truth of the propositions except in so far as the proof structures elucidate the relation of logical consequence that holds between the elements of the science and the phenomena it describes. So a proof for Euclid is not a demonstration of truth, it is a second-order demonstration of the knowledge we have of the elements of geometry as being the reasons why the things we perceive to be so are in fact so. This is why he doesn't annotate the text with the dependent propositions. It is also why the demonstrations do not deal exhaustively with all cases. For example Proposition i.2 which is 'To place a straight-line equal to a given straight-line at a given point.' This construction only deals with the case where the given point is not colinear with the endpoints of the given line. This is the only way the construction is used: it is never necessary to construct a straight-line equal to a given straight-line at a colinear point.

The idea is that one only understands the structure as a whole when one can see for one's self what are the connections between the proofs. Thus Euclid makes no attempt to exhaustively prove all the true Propositions. They are infinite and many have trivial proofs. The only propositions he proves are those that are part of the underlying structure that connects the generic classes of phenomena he wishes

[18] It is not clear this was Aristotle's choice of title. But even if it were, it should be appreciated how difficult it is to think of a title that will be immune to changing connotations of words over a period of twenty three centuries.

to explain. For example, 'angles in a segment', the peculiar species of angle which appear at the apex of triangles constructed from segments of a circle and are useful in the theory of figures inscribed in or curcumscribed by circles. This then connects to the theory of the area of circles so has a direct connection with the problem of squaring the circle.

Thus Euclid's *Elements* is an attempt to elucidate the underlying form of geometrical Truth which causes the true propositions to be true and all the others to be false. The intuition of the geometer is that this is real knowledge, i.e. that there is one true axiomatization of geometry, and that the process of proving propositions reveals this structure as a limit of the empirical facts of sense-perception. This is why empirical measurement is not allowed in Euclidean proof: if it were then geometry would not be demonstrating the reasons why empirical measurements are what they are, because it would be assuming this and would reduce to tautology. So in Euclidean geometry it is not permitted to use the compasses to transfer a length from one straight-line to another unless the two share an end-point and are thus both radii of the same circle. Thus the axiomatic structure of Euclidean geometry as a whole tells us the real reasons why the space we actually experience is three-dimensional.

More generally, the aim of any science, according Aristotle, is to elucidate the true reasons why things are as they appear and not otherwise. And the method should be the same: rather than attempting to produce indubitably true statements, the scientist attempts to capture the true structure of causality in that domain through the axiomatic structure of the definitions and postulates. An inevitable side-effect of actually achieving this would be that all scientific statements *would* be true, but we would never be able to know this. But aiming for the more knowable certainty is a mistake because it is easy to be certain simply by restricting the domain of discourse to just those facts that can be proved to follow necessarily from some fixed axioms. Those who attempt to do this however merely find that they end up arguing that ultimately the axioms are a matter of subjective opinion. Then the certainty they end up knowing is just the certainty of their personal opinion and this is a point from which no further knowledge is possible.

## 7 Scientific and mathematical knowledge

Aristotle held that we think that we understand something when we think that we know the reasons for it. So knowledge of some thing is what we demonstrate by explaining the reasons why that thing is so and not otherwise. Knowledge is different from art. A baker can make a loaf of bread but they don't necessarily know why they do what they do when they make bread; they just know that doing this reliably produces bread. One can show someone how to make bread and the method is *effective* because everything is tangible. One can point to the flour and the yeast and the water and show what it means to mix the dough and to knead it. Every important step is in terms of the objects of immediate sense-perception. Biochemistry on the other hand is not so easy to teach. But a biochemist should be able to demonstrate knowledge of why the baker does what they do when they make bread. The biochemist is in a similar physical situation the baker. The biochemist has a laboratory instead of a kitchen, and the machinery is more complex, but the biochemist is also working primarily with the objects of sense-perception. These may be in abstract terms of the readings from machines, but these readings relate to tangible physical specimens and they must know to which actual specimen the reading pertains to be able to interpret it correctly. So the biochemist and the baker are both human beings in a physical environment. But where the baker needs only to know the objects of sense-perception, the biochemist needs to learn to abstract from the particulars of sense-perception some *ideas* which are things the existence of which they have no *a priori* evidence.

These intangible things the biochemist studies constitute the knowledge of why it is that the processes the baker carries out produce bread. The biochemist can *demonstrate* the knowledge that constitutes the biochemistry of bread by showing how the abstract ideas correspond to the objects of sense-perception that the baker deals with. To do this the biochemist has to *define* the elements of the science: cells and cellular processes, which latter includes the elements of chemistry as well: molecules and the conditions under which they react together. The biochemist then uses logical deductions to demonstrate that the kowledge is complete by inferring the consequences of some premisses which are the observations made in the laboratory. The biochemist can explain the molecular reactions in the cells of the bakers yeast as they ingest sugar and release carbon dioxide, and how the molecules of starch in the flour are developed into gluten by the mechanical energy of kneading. Then how the yeast continues to ingest flour and produce carbon dioxide as the bread rises, then finally how the baking kills the yeast and develops the proteins into sugars which is what makes the crust. But why should the baker accept the biochemists explanation? The baker can't see that the individual cells in the brew of yeast are actually living organisms, nor does he have evidence that they consume the sugar he puts in, or the flour. He has at best indirect evidence that they do this because the yeast produces froth, and because bubbles are clearly observed in the crumb of the bread.

The molecular biologist draws a correspondence between the ideas of his science and the immediate objects of sense perception, but he cannot explain the *reasons why* the connection is as it is. This is just because this connection is not anything to do with biochemistry as such. Neither is it something a physicist could explain with physics, because there is equally nothing essentially physical about it. The connection between what is and sense-experience—i.e. the connection between the actual laboratory phenomena of any science and the ideas the science deals with—is the concern of a science Aristotle calls *first philosophy* which is the science of the first causes of all things: i.e. the science of *being*. This is not the science of being as an actual physical thing (which is physics) nor the science of being as a number (which is arithmetic) nor the science of being as an object in space (geometry). It is the science of being as just something that is.

This is the science that joins all the other sciences together through their *axioms*, which are the basic truths common to all. It cannot however explain anything essentially numerical, or essentially physical, or esentially biological, because these things are not explained by mere being, but but by being in some particular sense. By examining lots of philosophical and scientific knowledge of his day, Aristotle was able to identify only ten essentially different senses in which something could be said to be. These are called the *categories*. They are substance, qualia, quanta, posession, relation, place, time, state, action and affection.[19] These categories are the product of an essentially linguistic analysis.

Kant famously criticized Aristotle for not having a system by which he deduced these categories, and this shows clearly that Kant did not understand Aristotle be-

[19] Afection in the sense of being *acted upon by*, rather than of having a friendly disposition towards, some thing.

cause this is the empirical basis of his philosophy. If Aristotle had used some system to deduce the categories then they would have been determined by that system. But then the philosophy would not be an empirical science because the existence of such a system could not be something that is known intuitively. But these basic categories are known to us intuitively and this leaves open the possibility that we may discover some other language in which there are more essentially different ways in which something can be said to be. But it seems very unlikely this would be a human language, because we have no evidence whatsoever that any human languages are in fact essentially more or less expressive than any others. The underlying reason is most probably physiological: we share essentially the same physiology, and therefore we share essentially the same types of direct experience, and all languages will likely have ways to express those experiences, though some may be more or less expressive in particular areas. But this difference is inessential because ultimately we should be able to learn which linguistic expressions correspond in fact to which experiences. The reason we believe this is that our children apparently learn to do it, and furthermore that multilingual children have no problem integrating their conscious experience and describing it any of the languages they use. If there were in fact two fundamentally inconsistent human languages then we would expect to find this the cause of some kind of schitzophrenia in children brought up speaking both.[20]

Aristotle explains clearly that the nature of human reason is such that there is not just one 'sovereign science'. Science must be divided into separate subjects like physics, chemistry, biology, etc. and this is not just a useful convention that helps us to decide the division of labour: it is necessary if we are to know anything at all. This is because every science must take some knowledge as *basic truths* which are things the knowledge of which one cannot demonstrate. If we don't do this, then we can never demonstrate any knowledge, because we are caught in an infinite regress.

That this is in fact so is witnessed by the so-called 'problem of quantum reality'. Quantum mechanics is a supremely accurate physical theory which explains all physical measurements as being according to essentially stochastic mathematical laws with probability distributions which are determined by the macroscopic arrangements of matter and energy. But physical measurements are always realized in macroscopic arrangements of energy and matter. If we assume these macroscopic arrangements of energy and matter to be exhaustively described by physical measurements then the laws of quantum mechanics seem to explain all macroscopic phenomena, but only in terms of other macroscopic phenomena: the physical microscopic foundation simply does not exist because the individual events in which the macroscopic world is ultimately realized are random: there is absolutely no relation of cause and effect at the miscroscopic level. When we observe one outcome, why is that we do not observe a contrary one? The equations are silent: they state only probability distributions, not actual events; so what *actually* happens, happens just because that is what happened.

In the absence of an answer to this question it is very hard to say why it is that anything that we observe happens, yet it is said we can predict experimental measurements with extraordinary precision. The sense in which a prediction can be said to be

[20] *cf* Gregory Bateson's *Double Bind* theory of schitzophrenia which hypothesises that it can be caused by a child permanently held in a 'Catch 22' situation by a parent or parents who punish them if they do something and punish them, perhaps in a different way, if they don't do it. The child then develops two separate conscious selves, one which makes sense of one half of the contradiction and the other which independently makes sense of the other half. These two selves would be separate. They would coexist in the same brain but interact with the world through one body and thus could possibly learn to speak to each other through feedback from the shared motor system of the speech centre.

such implies that what is known in one sense is known first in a different sense. But if there is no sense in which there is a physical chain of cause and effect then what is the sense in which we know the predictions as being connected with the results they predict? In other words, without evidence of a causal connection from one event (the prediction) to the other (the predicted), what is the basis on which we can claim the science to be predictive? This is why the Copenhagen interpretation is unacceptable, because it asserts the reality of the knowledge and denies the empirical basis on which that knowledge could have been obtained. Thus quantum mechanics so interpreted is not science: it is merely metaphsycial speculation based on the assertion of a tautological relationsip that is a result of a formal mathematical theorem.

The laws of chemistry are the reasons why different types of molecules react in different conditions, and moelcules are macroscopic arrangements of fundamental particles, so quantum mechanics cannot explain the laws of chemistry without postulating the prior existence of molecules. And similarly biochemistry must postulate the prior existence of cellular metabolic processes, it cannot explain how they came from inorganic chemistry. The simple reason for this as we said: to explain something is to demonstrate the knowlege of it, which is to show that the knowledge is knowledge of the cause of that thing. And not everything can be demonstrated because all demonstration relies on prior knowledge and this leads to an infinite regress. So we have to start with something that is not demonstrated, but is instead defined or postulated. The only reasonable basis in fact for such a postulate is sense-perception.

This is the problem for physics: attempting to explain the macroscopic world on the basis of fundamental elements is impossible because as fundamental elements they don't have any internal parts so are essentially identical. The fundamental particles cannot therefore individually embody anything like the notion of perception which necessarily involves some perceiving whole which undergoes internal change as a consequence of the presence of the object of perception. It follows that perception can only be realized in terms of macroscopic arrangements of collections of fundamental elements. Therefore any explanation of macroscopic properties of collections of the fundamental particles *must* consist only of demonstration and must be an infinite regress because we have no grounds whatsoever to even postulate the existence of perception at that level. If this is unacceptable as an explanation, then the alternative is an infinite regress in *being* instead: that any fundamental particles are always in the end found to be composed of yet-more-fundamental particles.

As there is no Natural science that explains all others, there is no mathematical science that explains all mathematics. Mathematical logic is the use of mathematics to describe logic. Logical deduction is then reduced to a matter of calculation, the values computed being the truth values of logical (so in this case mathematical) statements. The truth is characterised mathematically as the set of true statements. Mathematical logic leads quickly to some deep problems. If the laws of logic are indeed laws then they are fundamental to all thought, including mathematics. Any mathematical logic is a symbolic representation of the rules of mathematical deduction. But it seems that mathematical logic is not able to characterize completely the set of true statements of mathematics.

Gödel's first incompleteness theorem shows that whatever formal system of arithmetic we define we can use the structure of the system itself to generate a theorem of arithmetic which, using some plausible assumptions about the consistency of arithmetic, we can informally prove is necessarily true, but which cannot be proved formally in that system. We can then take this *Gödel sentence* and define a new formal system with this theorem as an additional axiom. Then, using the new axiomatization we can produce another similar theorem which previously was not known to be true and could not have been proved formally. This process can obviously continue *ad infinitum*[21]. The set of truths that one produces in this is way is arbitrary: the proof of any particular Gödel sentence depends upon the particular choice of scheme used to encode formal proofs as numbers, and this can be any recursively definable bijection between finite strings of symbols and numbers. These truths are all therefore accidental: they don't depend on anything essential to numbers *qua* numbers (i.e. numbers construed as numbers rather than say, numbers as numerals in some particular representation.) It seems clear then that any recursive set of formal systems obtained in this way is going to contain many truths which, despite having been proved, are merely opinion: that of those who know them to be the Gödel sentences in some Gödel numbering of some system.

The problem with set theory when it is considered as a foundation for mathematics is that it pretends to be able to axiomatize all of mathematics. This a problem because what is a self-evident truth in one branch of mathematics is not necessarily self-evident in another. For example why is the principle of mathematical induction a self-evident truth of geometric topology? The subjects of arithmetic and geometric topology are separate because the domains of discourse are separate. Geometric topology may indeed treat features of manifolds as objects that can be counted, then the axioms of arithmetic can be used to prove theorems about those colelctions of geometrical objects considered *qua* numbers. But we do not need to import the whole consequential baggage of arithmetic, and why would we want to anyway, because it makes no sense to do that at all. The axioms of arithemtic are useless because they have nothing to say about the objects of geometric topology as topological objects with topological properties. In topology one orientable surface is not greater than another, nor is one a multiple of another. If we want to use arithmetic to reason about the genus of a surface we can define it as a number and then use whatever arithmetic we like to reason about these things as numbers.

Putting one set of axioms as the foundation of all mathematics assumes that the one set will accommodate all self-evident truths of any subject. This is certainly not self-evident because something self-evident is its own evidence, and the evidence that one set of axioms is good for all mathematics is surely something to do with all mathematics, and all of mathematics is not evident to anyone. The axioms need to be self evident otherwise we introduce inconsistencies. Witness the Banach-Tarski paradox which is a consequence of having a self-evident axiom of choice in set theory motivated by the existence of a self-evident axiom of infinity which was motivated by the need to model arithmetic in set theory. Then by representing geometry as sets of points we deduce that one unit spherical volume can be cut up into five pieces and re-assembled as two unit spherical volumes. Evidently then the notion of space being a set of points does not capture the sense in which volumes are partitions of space: the resulting 'volumes' do not behave as wholes. This contradicts the use of arithmetic to reason about disjoint volumes. But it is self-evident that disjoint volumes are wholes and should therefore be countable and amenable to arithmetic reasoning. For example if we want to prove the Kepler conjecture about the most efficient way to pack unit spheres then we need the spheres to behave as wholes, because the relation of relative efficiency in packing has no meaning if they don't. Then according to ZFC set theory, the most efficient way to pack $n$ unit spheres is to pack them into a space of one unit sphere.

[21]Trans-finitely so.

## 8 Truth

This widespread belief that the actual world is completely characterised by some symbolic description is not something that is a result of any sort of proof. Nevertheless, for many it is a very strong conviction. In *The Republic*[22] Plato defines the philosopher as one who comes to knowledge of Truth as distinct from any particular collection of beliefs or opinions. Now beliefs and opinions include those facts that are proved, whether by formal or informal mathematical methods, or by experiment. The assertion that the world is completely characterised by some symbolic formalism is the assertion that Truth can in principle be proved, and if that were so then Truth would be exactly the set of statements that could be proved true. Truth would then have been reduced to a matter of belief or opinion: that of those who know the proofs.

Aristotle was a student of Plato and they shared the same view of Truth as a whole, but Aristotle differed in that he did not see how the Forms, which include Truth, could have any existence in a world apart from the sensible substances and yet be the cause of those substances. According to Aristotle, the Forms are realised in our knowledge of substance: in other words, though the Forms are not sensible to us directly, they are known to us as the form of our abstract thought, conditioned by the attributes of substances which we can perceive directly. Thus all sciences including mathematics and philosophy are impirical sciences and rest ultimately on the actual substances in the world. Thus the basic truths of these sciences (the axioms and postulates) are known to us through intuition which is a mental process based on sense-perception.

For example, molecules are abstract, but we nevertheless say that they exist because they have definite measureable properties and combine in predictable ways known to chemists. Thus what is demonstrated mathematically (physical chemistry) and in experiments gives us a basis for belief in the existence of molecules. But at the same time we know that on nanosecond timescales chemistry is quite a different matter. The phenomenon of sonoluminescance seems to produce small areas of extremely high temperature, so the temperature of substances at different scales is not something that is explained by or explains all chemistry. And something similar happens in the temporal dimension: on nanosecond timescales 'non-standard' structure in molecules of water, for example, can exist for long enough to be detected. So whilst we continue to act as if we know what molecules are, we are also aware that we may have to change our ideas to accomodate what more we may learn. And we know we need to learn more because many things are not understood completely: for example it is not understood why water has many of the curious properties it does, nor is it understood exactly how protein molecules fold up into their characteristic shapes.

The idea of Truth then, is just the intuition that within the particular demonstrations and proofs which give rise to our particular beliefs, there is a universal; extremely abstract and more or less vaguely perceived, but nevertheless real, and not merely an artefact of either the order in which we discover things or the particular language we choose to describe them. The intuition is a strong one because we find we can teach others about these things and whatever culture they come from, whatever language they speak, and whatever order we explain the phenomena, these others can come to know these same things and then on the basis of this knowledge they can discover more, and that what they discover can be communicated back to us.

[22] Chapter XIX, v. 474 B-480

So the knowledge we share is not just knowledge of what has been demonstrated or proved: it is something abstract from this because the particulars being demonstrated and proved are constantly in flux.

The all too common error is to mistake the Truth for that certain knowledge which we establish through demonstration or proof. This reduces truth to *sophistical knowledge*, the mere belief in the appearences of things. This is the error that is made in the philosophy of mathematics by both the logicists (Hilbert, Russell, *et al*) and their critics, the intuitionist logicians (Heyting and others.) Both of these movements sought a foundation for mathematics in the certainty of proof. It is the same mistake that was made in the philosophy of science by the Logical Positivists Wittgenstien, Carnap *et al*, and also by Popper, their arch-critic. The logical positivists sought to banish 'empty metaphysical statements' and build a foundation for real scientific knowledge by defining the language of science to restrict it to just the statements that could be experimentally verified. Popper is considered to have comprehensively demolished any hope of this by arguing from the fact that no universal can be certainly verified in experiment. But Popper nevertheless maintained that scientific knowledge was real, but he ultimately failed to characterise what a scientific statement actually is because he too focussed on the truth or falsity of individual statements. He had hoped to characterise science as the statements that could be actually or potentially falsified, but ran into the problem that in practice all scientific statements are statements of probability, and no measurement of a value described by a non-degenerate probability distribution could be certain to constitute a falsification of a theory because it could always have been produced by chance.

Despite this failure Popper always maintained that scientific knowledge is nevertheless something real because, he said, it would be ridiculous to deny that the atomic bombs that the United States of America dropped on the Japanese cities of Hiroshama and Nagasaki at the end of the Second World War were real. But many technological devices have been successfully built on the basis of partial or unsound knowledge. It is true that the knowledge that lead to the search for the theory of atomic energy was knowledge of sub-atomic particles and therefore something strictly beyond experience, however there was plenty of laboratory evidence that atomic energy was real enough, and all through the development of the project there were physical experiments that kept it on track, some quite evidently releasing astounding quantities of energy. Theoretical physics did not, out of mere theoretical speculation, produce some equations which engineers realised in two devices which worked first time. Though the bombs were undoubtedly real, it does not follow that each and every statement on which their operation was explained was true. No one knows this better than experimental physicists. I submit that no-one has a stronger intuitive sense of the reality of physics than these very same experimental physicists who experience it immediately as force, sound, and heat and light. It is not the existence of the real that needs to be argued, just the statements we make about it.

# 9 Language and logic

Language had been used to articulate reason for millenia before Aristotle formalised logic. Nevertheless nowadays we find it quite sensible to ask the question 'What came first, logic or language?' Asking the question presumes that one or other is prior. I would guess that most modern logicians would say that the answer is 'logic', and the reason would be that it is far from clear how language could exist in a world which did not obey the laws of logic, whereas the laws of logic apply without language: if $A$ is a statement of fact and $B$ is a statement of fact then surely it is also a fact that 'The statement "$A \wedge B$" is true if and only if $A$ is true and $B$ is true', and whether we can chatter about it or not makes not the slightest difference. The fact that the phrase '$A \wedge B$' is a statement in some language is incidental: what it *denotes* is a situation in actual fact which is in no way dependent upon whether we know it or not.

In modern logic we typically start with a formal language described syntactically and then define truth in that language in the form of a model described in some other language, the semantics of which are more or less clearly known. The model is supposed to be a description of what we mean by the informal term 'the facts' and this is why it can serve as a definition of truth for the formal language. Endowing a formal language with a model fixes the interpretation and so fixes the set of true statements. Thus in symbolic logic it is possible to prove for some languages that the set of statements that can be proved in some formal proof system is exactly the set of true statements according to a certain model.

The difference between Aristotle's logic and modern symbolic logic is that in Aristotle's logic one starts with verbal discourse of the evident facts (the premisses) and the logic is a complete enumeration of all the forms of discourse of whatever other facts (the conclusions) are necessarily evident in the light of these. The fact that the logic deals primarily with verbal discourse is important: a discourse is an actual act by some conscious being and therefore the true statements are defined as those judged to be evidently true in the act of making them. Thus truth cannot be proved in a formal proof system because the forms of statements are not enough to to determine their truth: two statements at different times may have different truth values but the same verbal form. For example 'there was a fire in town yesterday' could be stated by two different people, one of whom witnessed it and the other who read it in a newspaper without noticing that it was six months old. In Aristotelian logic it is the discourse that is known to be true or false, not the form of the words.

The logic of Aristotle is not 'the laws of thought' as many of the nineteenth century logicians conceived symbolic logic. There are no laws of thought as such: there is just reality, and what we think about it is pretty much up to us. If we want to know the world around thus though, then we need to make sure our thought is in line with reality. We say we understand something when we know the reasons for it. So we would like to know the reasons for what we see. But we do not perceive reality very accurately. The procedure by which we learn is to observe just what we can and draw whatever conclusions necessarily follow from this. This is the role of formal logic: to allow us to infer the necessary conseuences of the observed facts. These conclusions may not all be immediately observable, but if any of them contradict what we can actually observe, then we know that at least one of our observations was not correct. This can only be because we have not described correctly what it is we observed. So we know that we must modify one or more of our definitions which are the ideas we have about what are the actual things we are observing.

The true and the false are characterised by conjunction and separation. Only statements which have the form of propositions can be judged to be true or false and a proposition is the assertion or denial of some predicate of some subject. The sense in which a truth is a conjunction is the sense in which the discourse is one with what we experience. So as we look at a red flower 'the flower is red' is what we see, but 'the flower is not red' is other than what we see. So the former is a truth and the latter a falsity. But 'the flower will wither within 24 hours' is neither because the subject is the state of the flower at various times over the course of the next 24 hours and that is not something that can be the subject of experience until at least 24 hours have passed.

Discourse can only be judged true or false once: at the time it is made. If at the time the discourse occurs it is not judged to be either true or false then it is called 'indefinite'. This point is frequently overlooked, possibly because it seems to contradict the law of the excluded middle or the the law of non-contradiction, but that is not so. It is explicit in the definition that we not assert the existence or non-existence of the subject. So in defining the true and the false we do not commit in any way to deciding the truth value of any statement concerning something indefinite or non-existent. In effect there is one truth predicate for each category, and only that of the category of substance asserts the existence or non-existence of the subject.

The true and the false are defined this way because the universe is not deterministic: actual things can undergo change. It is clearly possible then to make meaningful statements about undetermined events, but these statements then cannot be judged either true or false at the time they are made, because if they were then the events they refer to would be determined. Such statements are not useless because they may be the consequences of knowledge which is believed true and they can be made the premisses of hypothetical deductions which explore the consequences of that knowledge.

The empirical sciences and the dialectic deal with belief: the empirical sciences with the belief in what has been proven and the dialectic with belief in general, whatever its basis. One cannot believe in anything more than one believes in those things which one knows, and the pure sciences produce unshakeable belief: witness mathematics. Aristotle's formal logic is the logic of belief. But unshakeable though it may be, belief could always turn out to be misplaced. This is not a contradiction because when we understand that our belief was misplaced we understand not only the new beliefs but also the exact reasons why the old beliefs were wrong. The new beliefs therefore constitute a better knowledge of the world and if the old beliefs were unshakeable then new ones are at least as unshakeable, but possibly even more-so because there are statements which contradict the old beliefs and do not contradict the new, and if the new beliefs include a complete undersanding why the old ones were wrong then it could be than any statement that contradicts the new would also contradict the old.

Now what we should believe are just the things we know to be true and these are the things we can prove mathematically or in physical experiments. These things we call *the true*. That whole called Truth on the other hand is not any of these things which we know through proof: Truth is what compells us to change our beliefs, even if they are the unshakeable conclusions of proof. It is Truth that is the limit of the process of scientific knowledge, but no collection of particular truths will constitute certain knowledge of Truth. Truth is something which we come to know not through proof itself, but through intuition: the same means by which we come to kow the axioms of any particular science. The existence of Truth is the axiom of philosophy.

## 10 Aristotle's theory of knowledge

We only know that we know something when we can describe it to someone else. Anyone who claims to know more than this will not be able to describe what it is they know, so will not be able to demonstrate their knowledge. Thus we know that all knowledge is described in words. Words are sounds that correspond to thoughts. Written words are symbolic representations of sounds. Though all the sounds are different, and though we may use different words to describe the same thing, the thoughts to which these equivalent descriptions correspond are identical. This is why we can describe what we are thinking to other people. [23]

Now because it is described in words, which words must have known meanings, all knowledge is inseperable from the means by which we obtain it. For example, to know the meaning of the phrase 'the charge on an electron is $1.602 \times 10^{-19}$ coulomb,' we need to know at least what the words 'charge', 'electron' and 'coulomb' mean. The word 'electron' means different things in different contexts. The best way to describe what an electron *actually is* is to describe the physical experiments that reveal the presence of electrons: these exhaust the possible senses in which the electron can be said to be. Similarly 'charge' has different meanings: the charge on a macroscopic body could be described as being due to an excess or deficiency of elementary charges of some kind, but the nature of the charge on a single electron certainly cannot be explained in this way, so although it seems to be the same idea it is in fact another concept altogether, but having the same name. Thus the phrase as a whole could be used in at least two different senses, with a different meaning in each. In one sense it could be an assertion of fact, and in another the conclusion of a deduction based on demonstrative premisses, perhaps the statement of observations in an experiment.

But having described these experiments using other words, we necessarily must know what those words mean, and these words in turn will have meanings that are known only in terms of other words, and so on and so forth. This process must come to an end if knowledge is to be said to have any foundation in fact at all, and indeed it does.

## 11 Inductive and deductive inference

Ultimately all knowledge rests on the means by which we are able to learn to use words to make and understand descriptions of sense-perception: i.e. direct actual experience. This is achieved in the mental process Aristotle calls *induction*. As Aristotle uses the term, induction is an immediate judgement and not, as sometimes claimed, a gradual realization arrived at by observing many different instances. In Aristotle's words 'Induction is the perception of a universal in a clearly known particular.' For example, on observing a particular pair of mice in captivity and discovering that they mate and produce litters of baby mice, it is immediately apparent that all mice reproduce this way: we do not find it necessary to observe many different pairs of mice doing the same thing to be sure that it is universal. Induction is not infallible: a child might learn by induction first that all women are called 'mummy', and only later refine the concept. In the process she learns simultaneously to discriminate between other women and her mother: i.e. between women who are mummy and women who are not-mummy. In science we use induction to infer that the results observed in the laboratory apply in all situations where the necessary conditions obtain.

Note that we perceive the generality at one and the same time as we perceive the

[23] Note we are *assuming* that we do indeed denote the same thoughts when we give equivalent definite descriptions. We cannot consider the possible consequences of mistaken belief, whether accidental or through being deliberately misled, because this is not something that is amenable to reason. However the various means by which people can come to be misled in the first place are amenable to reason and this is what Aristotle explores in *Sophistical Refutations*.

more clearly known particular. The child who learns to identify her mother with the word 'mummy' learns this at the same time she learns the vague complex of motherly sensations to which the word corresponds. And when she later refines the notion of her mother, she does this at the same time as learning about the other people who are not her mother. But these others were not known before except as some less-clearly perceived whole she called 'mummy' which in fact included her actual mother. Similarly the abstract complex we call 'the laboratory conditions', are perceived as the same as those which naturally occur. This is just because that is how we intentionally contrive them to be. When we later discover some reason why those particular conditions are not universal in the sense we earlier believed them to be, then we know that we have discovered something new about the Universe. For example, that the strength of the gravitational field affects the frequency of radiation emitted by an accelerating electrical charge. This was something that was not anticipated and which we did not detect in the laboratory because the effect was not evident in the particular conditions we contrived: the gravitational field did not vary that much.

We obtain knowledge by two essentially different means: sense-perception, also called intuition, and deduction. Sense-perception is fundamentally a process of perceiving wholes. Take for example a rock that I have on my desk. The rock is a whole because of the electrostatic forces that bind the molecules together. We can formally characterise the forces and the molecules, but these do not explain the rock, because the rock is a particular rock and is only that particular rock because of its history: all these molecules of sulphur and calcium etc. have to be exactly where they are now for the molecular forces to have the effect of holding the rock together. So the molecular forces and the nature of the individual molecules do not explain the rock because they do not explain how the molecules came to be where they are now.

The explanation of the rock lies in the context of the wider world: it came from somewhere near the summit of a dormant volcano called Acotango, on the border of Bolivia and Chile. The concentration of sulphur presumably came about because of sulphurous gas emitted from the volcano which crystallized as at cooled. This wider context includes other things that we know about it, such as that is the particular piece of rock that I brought back from the mountain in April 2010, etc. etc. All of this information together identifies this particular rock. If it were not for this wider context the molecules of the rock would be scattered about and would not affect each other as they do in the matrix of the rock: the collection of molecules, even if we could identify each one individually, would not be anything that could be known as a rock because it would not have any of the attributes of a rock. It is in this sense that the rock is a whole, or what Aristotle calls a *primary substance*.

Now anyone understanding these words of mine can imagine the rock I am describing and so, if you believe me, the phrase 'the rock' denotes a primary substance, but as far as you know it, it is essentially different in that you know it only as an 'object of thought'. But the rock as an object of thought is the same rock as the object I can pick up now. If it were not I would not be able to form the intention of picking it up to examine it. So for me the rock as an object of thought is one and the same thing as the actual rock of my sense-experience. Thus I judge that the rock is a true rock in the sense that it actually exists, and I am able to make true statements about it, e.g. that it is yellow, crystalline, sulphurous, semi-translucent, about 3 cm × 1 cm × 2 cm etc.

Now the rock is quite soft so I could easily crush it into a powder. This I could mix with some saltpeter and carbon which, if the rock is indeed pure sulphur, I could then burn, producing a great deal of heat, light, smoke, evil-smelling gas and leaving behind a residue of ash. The rock would then no longer be, but the rock as an object of thought would continue to exist after I have burned it up: in principle anyone could read this text I am writing now and so at any time in the future the rock may exist as an object of thought in exactly the same way it exists as such as I write. Thus the rock as an object of thought has no fixed relation to time: though it came to exist as an object of thought at the time I picked it up, thereafter it has no particular relation to time, so Aristotle would say the rock as an object of thought is eternal. The rock as an object of thought could not even be destroyed by being completely forgotten: to see this just consider all the other rocks as objects of thought that have been forgotten: you will find that you cannot. In other words, to know that something was destroyed we must know that it had previously existed, and if we know the rock as an object of thought had a previous existence then we know it as an object of thought and so it cannot have been destroyed. This is because we only know something as having been forgotten when we recall it again, and so everything that is ever known to have been forgotten is in fact known. So objects of thought either exist or they do not and they pass in and out of existence completely unexpectedly; we can never catch them in the act of coming to be: they either are or they are not and they are never anything in-between.

The objects of thought are not tied to the present or the future. Had someone predicted that I would collect this rock on that date, then the rock would have existed as an object of thought before it became known to me as an actual rock. As an object of thought existing before the rock itself, it would not be particularly well-known: many details may have been omittted, such as its size, its composition etc. but we could nevertheless know them to be the same because that is the necessary condition which means that my collecting the rock was something that was successfully predicted.

Objects of thought are not patterns of electrical activity in individual brains. An object of thought is the correlative of all the individual thoughts which are about that thing. The objects of thought are each unique so that when I mention Socrates as the verbal correlative of my thought it is the same object of thought that occurs to you when you interpret my utterance. Though we very probably have different sets of knowledge of Socrates, we are nevertheless thinking about the same man. And this is so even if your cat happens to be called Socrates because the context in which I mention Socrates makes it clear that it is the philosopher to whom I refer, not your cat. Despite being dependent upon the meanings of words, true knowledge is not relative. Rather it is absolute because, even though we could be mistaken about the meaning of those words, all true knowledge is instantiated in actual, definite experience.

All objects of thought as such are eternal and this is the foundation of reality: the objects of thought together are the sum of human knowledge. Thus we might imagine human knowledge is a vast network of interacting objects of thought, popping in and out of existence according to who is thinking about what. If we pay close enough attention to what we know then we may get a glimpse of it, but the picture any one of us sees will always be a static network of objects of thought, eternal and unchanging. No one of us will ever catch it in the process of changing, because to perceive change we must perceive a difference between 'before' and 'after' and, as we explained above, we cannot do this when we consider only the objects of thought because they are independent of time.

## 12 Essential attributes

An object is a particular object because it has some specific features by which we identify it. These are all the things essential to the being of that object; attributes like form, colour, etc. Accidental attributes are those that are not part of its identity, for example its location at a particular time, or if it's a vertebrate animal, the fact that it may be sitting or standing. The essential attrbutes of objects that we know directly are not difficult to ascertain. But the more abstract the object, the more difficult it is for us to perceive, it being that much further removed from sense-perception. The aim of science is true kowledge, which will always be expressed in terms of differentia related to sense-perception. It is through applying these differentia to the particular objects of sense perception that we obtain knowledge of the more abstract things which are not directly known to us. Thus the distinction between accidental and essential attributes is crucial because it is only through the essential attributes that we have reliable differentia, and it is only through their differentia that we come to know these abstract things.

Primary substances as objects of thought are eternal and unchanging. Actual primary substances themselves, as objects of sense-perception, are capable of undergoing internal change. This is what underlies our ability to make inductive inferences: the clearly known particular lies within the less clearly known generality, as a part within a whole. Each particular is percieved as a definite universal within that generality. For example, it is perceived immediately that the experimentally observed magentic flux density obtains around all moving charges, wherever they are and whatever their state of uniform motion. As we learn more such particulars we obtain a sharper concept of the generality, which is the phenomena of electrodynamics as a whole.

The Scottish philosopher Hume famously deduced that we could never know anything by induction. His error was in taking the knowledge we have of the world to be independent of the means by which it is obtained. In this case induction is reduced to a probabilistic process which can never produce certain knowledge. Thus Hume claimed, by way of example, that no matter how many swans we observe we can never know that all swans are white. This is because Hume took the 'objective fact' of all swans being white or not as being independent of our knowledge of swans. This is patently absurd, because a bird is a swan only because it is identified as being of the species. But a bird is a primary substance and a species is not a primary substance: a species of animal does not have an existence apart from the actual animals that are characterised by those definite attributes by which we know it. Thus if we know a particular animal is some particular species we know what are those definite attributes. Hume's example would have been better stated as the impossibility of knowing that all swans are large white birds that have two wings and two legs and a long neck and which nest near water on which they raise their young. These are all essential characteristics of swans as we know them. They are reasons why a particular bird is a swan. It is not only ornithologists who know this: the swans know it too, so a swan which is the size of a sparrow, bright purple, with only one wing, and which burrows into a mud bank to build its nest will not very easily find a mate amongst swans. So being white, having two wings etc. is the reason why a particular bird is a swan, and so it is also the reason why we are correct in our characterisation of the species 'swan', i.e. that we have found the true definition of the species. The inductive inference that all swans are white is then immediate.

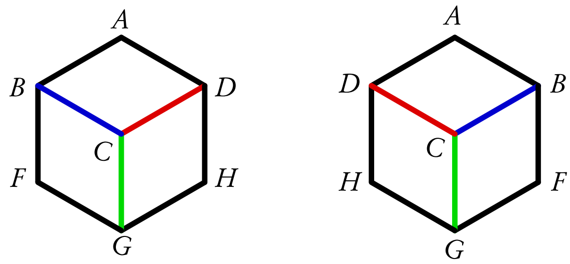
The sentence 'all swans are white' has a meaning which depends on the meaning of the words 'swan' and 'white'. For the former to be a useful word it must identify a species uniquely within a genus. That species will have what Aristotle calls essential attributes, and these will obtain universally, i.e. in all members of the species. Of course this does not preclude the possibility of a hatchling in a swan's nest being black, but to know that it is in fact a swan we need to know that it was from the egg of a swan. We know this when we know that both its parents are swans, which is something we know from the definition of the species. Then we know immediately that the black signet is atypical and that therefore there must be some accidental cause, and so the explanation for the blackness will not be found in anything we know about the essential nature of swans. It may turn out that this black swan finds a mate amongst the white swans, and that some or all of their offspring are black. The result may be that the species evolves and so the statement 'all swans are white' would then no longer be true. We would have re-defined the species, having learned that whiteness is not an essential attribute of swans as we thought. However this would not mean that the statement could not have been judged true by Hume when he uttered it.

Things change. Thus the same statement may at one time be true and at another false. But this does not preclude the possibility of certain knowledge because certain knowledge is personal and immediate to some knowing subject: when I say that it is certain that all swans are white, it is *I* that has this certain knowledge. When I make the declaration 'I am certain that all swans are white' I make a statement and I stand behind my judgement of its truth. This is a definite act and it occurs at some time and some place.

In Aristotle's sense of the term, a species is something distinguished abstractly within a genus by differentia which are predicates which uniquely identify that species as distinct from all the others of the same genus. So there is never any doubt that a particular species is actually a whole species and not a sub-species or a mixture of two or more true species: the definition of a particular species is always a species as an object of thought. Aristotle did not consider species to be primary substance, so there was no possibility of a species evolving. Of course it is possible that the knoweldge we have of the actual species is not true knowledge, but that is what we are trying to determine. We do this by assuming the knowledge we have is true and exploring the consequences. For example, it may come to pass that, having observed several black swans, I will at a later time stand behind the contradictory statement 'I am certain that not all swans are white'. Both of these are meaningful statements and both are a useful basis from which to reason further and learn more about the world. Indeed, had I not at one time known that all swans are white I could not be certain at the later time that some swans are not white, because the so-called 'black swan' may have been a result of a cross breeding with another species called, say, 'Australian black swan', and as such would not be a swan, but a hybrid. It would then still be the case that all swans are white.

## 13 Formal logic and symbolic logic

We perceive wholes, and this is the basis of certain knowledge, but in general, purely symbolic characterisation of a whole is impossible. To see this consider two cubes with vertices labelled $ABCDEFGH$, but where one is the mirror-image of the other, so that left and right have been swapped:




Now consider the cubes represented symbolically. One way to do this is to describe each as a graph: we give just the set of pairs of vertices which pairs represent the edges in terms of the two vertices they connect. So we may symbolically describe the first cube as the set

$$\begin{gathered}\{\{A,B\},\{B,C\},\{C,D\},\{D,A\},\{E,F\},\{F,G\},\\ \{G,H\},\{H,E\},\{A,E\},\{B,F\},\{C,G\},\{D,H\}\}\end{gathered} \tag{1}$$

and the second as the set obtained by performing the reflection, which is the permutation which in cycle-notation[24] is written $(BD)(FH)$. This is a formal description of both cubes, but it fails to distinguish the two because the second set is just

$$\begin{gathered}\{\{A,D\},\{D,C\},\{C,B\},\{B,A\},\{E,H\},\{H,G\},\\ \{G,F\},\{F,E\},\{A,E\},\{D,H\},\{C,G\},\{B,F\}\}\end{gathered} \tag{2}$$

which is the same set as describes the first cube. The two cubes however are plainly different and we can perceive the difference immediately because sense-perception perceives wholes.

There is nothing mysterious about this ability of sense-perception: it is just the plain fact that any formal representation is interpreted as an act, i.e. in actual fact and if a formal representation has dual interpretations in actual fact, then since actual fact is one or the other of the pair of duals, actual experience will have a basis on which to discriminate between the two. But this commitent to one or other is something that occurs only in actual interpretation, so a purely symbolic formal characterization cannot make the distinction until it is interpreted.

To see this, imagine that you have some written instructions which tell you how to make one or other of these two objects from distinguishable parts. The instructions tell you all the relations between the parts such that if you ensure these relations hold then you will have constructed the figure. Your mirror-image interperets left and right dually, and so given the same symbolic instructions your mirror image will contruct the other figure, therefore the symbolic representation does not distinguish one from the other. It cannot distinguish left and right because what we call 'left' and 'right' is solely a matter of convention: so it cannot be deduced as a consequence of anything that does not already contain within it the same information, such as clockwise/anticlockwise, inside/outside, forward/reverse, object/image etc.

The distinction between left and right handed geometry has practical importance. Certain molecules that have significant metabolic roles in living organisms occur in stereo-isometric forms and the left and right-handed forms do not react in the same way because the organisms typically express a bias towards one form or

[24] Each pair is a transposition cycle, so $(AB)$ for example, means 'swap A and B'. Cycles are composed by acting in order from left to right, though that is not significant in this example because they are pairwise disjoint.

the other; presumably this is an hereditary trait. The distinction is also important in physics because certain experimental measurements can distinguish left and right-handed 'space'.

It is not only sight which perceives wholes. The sense of touch distinguishes one from many directly, as you will experience if you cross your fingers and move them across a textured surface: the immediate perception is of distinct objects. From this we can deduce that normally when we touch a surface with our fingers we perceive it as one, not because we deduce that it is one from some other evidence, but because we directly experience it to be one. Neither is this to be considered mysterious: it is just the fact that you are one whole being and that your sense-perception is integrated from all the individual stimuli. This is absolutely necessary for you to be able to function in the world because it is only when the senses are integrated that they can be said to be informed: when you feel your way around the dark, say, you integrate your sense of where your limbs are in space with your sense of touch and hearing. It is only the whole of the immediate sensations that you experience as, say, standing on sloping ground with an uneven wet surface on your left, and falling water on your right. Experience is always experience of something because it is composed of repeated memories of sense-stimulus. Without this integration the sense-stimulus have no meaning.

We need to reason about such formal distinctions logically, therefore logic must be able to accomodate this kind of reasoning. So we must draw a distinction between a formal representation and a purely symbolic representation. A formal representation is a symbolic representation *actually interpreted*. The symbolic representation on its own cannot be said to have any particular form until we know how to interpret it. For example, we need to know that the letters represent vertices of a cube before we can interpret the symbolic representations in (1) and (2) as a cube, and we need to know the *sense* in which, say, the order of the vertices written $ABCD$ trace the edges: is it clockwise or anti-clockwise when viewed from outside the cube? Both of these things constitute knowledge which is not, nor could ever be, inherent in the symbolic representation.

Thus we distinguish formal logic from symbolic logic. Formal logic can be represented symbolically as symbolic logic, but it is only relevant to scientific knowledge in the act of being interpreted as being about actual experience. Aristotle's logic is formal logic.

## 14 Definition and meaning

Aristotle frequently clarifies linguistic conventions. For example he discusses the different senses in which people use the term 'possibly' and then he takes into account these different senses in proving the validity of deductions involving possibility. But he very seldom defines the meaning of words. The distinction between stating how words are used and defining the meaning of words is that the former is formal and the latter merely symbolic. When we state how a word is used we interpret it in actual experience, but when we define the meaning of a word we use it as a symbol standing for other words. Of the definition of the meaning of a word one may always ask for the meaning of the words in terms of which it is defined. In stating how a word is used however, we interpret the word in actual experience and so we take it as understood what the meanings are in the enumeration of the different senses in which the word is used. For example, we earlier noted that the word 'charge' is used in two different senses when referring to fundamental particles and when referring to macroscopic bodies, we did not define what those were however; we took it as understood. We cannot define any one word as having two or more meanings because then every statement we make using it would be ambiguous. Analysing the different senses in which the same word is used is a matter of deciding the meaning of the word in the context in which it is actually used, and this is quite a different matter to defining the meaning of the word itself. If we had wanted to explain the distinction between the charge on an electron and the charge on a macroscopic body we would have described the experiments that determine the electronic charge, and those that determine charge on macroscopic bodies. These descriptions are definitions of charge as used in the respective senses: i.e. we are defining charge itself and not the meaning of the word 'charge'.[25]

The meaning of a word is something that is determined as much by the context in which it appears as by the word itself. For example the sense in which we earlier used the word 'Socrates' was as referring to the philosopher as opposed to a putative cat, and this was supposed to be clear from the context. The context is the context in which the word or phrase is interpreted and interpretation is the ground of formal logic. Thus in formal logic we are concerned mainly with the sense in which words are used and barely at all with what they mean.

This distinction is subtle and all too often missed. As a result a great many philosophical discussions end in pointless circular reasoning concerning the definition of the meanings of words. This does not happen when we take it as understood that we know what words mean in the particular senses in which they are used. The problem comes when knowledge is considered to be independent of the means by which it is obtained, because then we start to consider meaning as being extensional rather intensional. But what someone means when they use a word is exactly what they intend it to mean. This is why we can name our pets after dead philosophers without expecting any particular ill-effects. It is also why saying that 'I say what I mean' is the same as saying 'I mean what I say' whilst saying 'I eat what I see' is clearly not the same as saying 'I see what I eat'. Because definite utterances signify definite thoughts, and the thoughts to which definite utterances correspond are identical for all concerned: aside from ambiguities, it is impossible to mean what you say and yet not say what you mean, and *vice versa*, you cannot say what you mean and yet not mean what you say. This is also the explanation why it doesn't much matter what we call things, as long as we know what we are talking about. For example I may choose to call that a purpose which others would call a porpoise. I may be inviting confusion or unnecessary argument, but provided I know what I *mean to say*, either can be explained away[26].

## 15 The true and the false

Since the same statement may be at one time true and at another false, we must discriminate between an *utterance* and a *proposition*, which is a form of utterance. By utterance we mean a conscious speech act by someone at some particular time and

[25] The fact that electrical charge is apparently two different things is a problem with physics, not philosophy. There are two different phenomena because the charge induced on a macroscopic body depends on the type of material that is used to induce the charge. The magnitude and polarity of electrical charge on a macroscopic body is accidental, but the charge on an electron on the other hand is an essential atribute of the electron.

[26] Usually by appeal to a purpose.

place. A proposition is the assertion or the denial of some predicate of some subject. Thus if the utterance has the form of a proposition then we say it is true if it asserts of what is so that it is indeed so, or of what is not so that it is indeed not so. And if it asserts of what is not so that it is indeed so, or what is so that it is indeed not so, then we say it is false.

Not all utterances are definite because it is posssible to assert or deny some particular predicate of something indefinite: for example I might state that I am going to work tomorrow, but it is possible that I will die in the night. The subject is 'what I am going to do tomorrow' and this is indefinite: whatever I could do tomorrow, it is also possible that I not do that thing. Therefore there is no basis on which to judge the truth of the utterance. Because a particular utterance is an act and not a substance, it exists only as an object of thought and therefore is eternal and unchanging. Thus someone, on seeing me working the next day may assert that 'yesterday he said he would work today, and indeed he is working so, *with hindsight*, what he said *was* actually true' but this is only a manner of speaking, and in fact it does not make the first utterance true. The first utterance remains indefinite because it is only the actual utterance that could be said to be true or false, not the proposition: the same proposition could be the form of another utterance which is contrary to the first—i.e. made by some one who did not manage to work the next day. Thus no prediction in itself constitutes knowledge because a prediction is always indefinite: what constitutes knowledge is the knowledge obtained with hindsight of whether or not the prediction was in fact correct.

In mathematics and logic, knowledge is exact because the knowledge in mathematics and logic concerns the forms of things and whenever we can meaningfully speak of these, they are well-known. But not everything can be said to be known exactly. Whenever we are dealing with matter we do not have exact knowledge. For example if I were to state that 'the length of the table is 1.7 m' then, though '1.7 m' may be definite (for example, by the SI definition of the metre), the length of the table is not definite in the same way. To know the length of the table I need to measure it, and this will always be by some means that is an indirect comparison of the table with a physical realization of a metre. In the case of the SI metre, the physical realization used to be a particular bar of aluminium at a certain temperature, but now it is the distance that light travels in around $1/300,000,000$ s. So 1.7 m *is* the distance light travels in an interval of time 1.7 times longer than this. Suppose I set up a 'light-ruler' to realize the distance of 1.7 m. This could be some sort of a device with a pair of prisms at each end, a laser light source, a 'caesium fountain' atomic clock measuring the interval of time, some light sensors to measure the arrival of the laser light at each prism, and a mechanical means to precisely adjust the distance between the prisms to match the interval measured by the atomic clock. This comparison would be done electronically, probably using a pair of 'logic gates', one to detect whether the measured time interval ended before the light arrived, and the other to detect whether the light arrived before the time interval ended. The distance between the prisms would then be adjusted so that neither of these of these detectors were triggered.

Now the distance between the prisms *is* the distance 1.7 m, by definition. But this is not the same thing as the table, so to know the length of the table is 1.7 m I must make some kind of physical comparison between the two distances, i.e. I must experience them to be the same. Then, because I also experience 1.7 m to be the same as the distance between the prisms (because that is what we define 1.7 m to be) I will experience the table to be 1.7 m and so the utterance I make to that effect will be true. But in fact I cannot ever experience them to be the same because they are two distinct substances: one is a table and the other is a complicated piece of apparatus. The best I can do is to make sure I don't experience one to be different in length from the other. Thus I cannot ever have evidence that the table is 1.7 m, so I can only truly state the absence of evidence to the contrary, and I can make it clear what the significance is of this by stating how it is that I attempted to detect that the table was not 1.7 m long. This gives me a way to state the possible error in my measurement, because I can take a similar approach to my measurement of the time interval by analysing the way the caesium clock and its associated intrumentation work and thereby get an idea of the error inherent in the measurement of the time interval that I compare to the interval the light takes to travel between the sensors. These sensors and the logic gates etc. will also have knowable errors. Then there are errors due to the change in temperature and the gravitational field, and perhaps movement of the apparatus. These too can be quantified and incorporated into the measurement error. The result will be that my statement of the length of the table is indefinite, but in quite a precise way. This is not knowledge though, because it is indefinite. The only use to which this precise statement can be put is in evaluating the significance of other measurements. These will also be indefinite in the same way, however. So in the physical sciences we work only with indefinite statements: things known more or less precisely, but never completely. Statements in the physical sciences are not therefore true or false: they are always indefinite.

## 16 Reasoning about the indefinite

If we know the reason why a particular prediction was made, then the knowledge of whether or not it was indeed correct is knowledge about the cause of the prediction being what it was. For example if we know that a particular species of flower occurs in two colours, and that in some particular region we have reason to believe that most of the flowers of that species are the same one colour, perhaps because of some environmental characteristic of the region, then we expect to find this to be the case wherever we look in that region. When we see what we expect we consider this to be a verification of the knowledge, but if we observe that in some small area within the larger region there are the same number of flowers of each colour then this does not mean that knowledge of the region as a whole is false: it could be due to chance, or else there could be a specific reason, i.e. a cause, so we should look for some condition which distinguishes this area from the rest of the region. If we find the cause then we have refined the knowledge we have of the region and the flowers. This new knowledge will then be the basis of new predictions which we can expect to be more accurate over the region as a whole. So a discrepancy between expected and actual probability distributions is a sign that we may not know everything that is going on, and that we should look for reasons for the discrepancy. On the other hand the absence of discrepancy is a verification of the knowledge on which we based that prediction.

So in summary, any statement about the future is indefinite, but if it is based on knowledge about how the world is then we can learn something by observing the prediction to have been incorrect. What we learn is that the knowledge on which we based the prediction is incorrect. This should prompt us to try to learn the reason why.

But if all the particular statements we make are indefinite, then we need to explain how anything said in the physical sciences can be known to be true or false. In fact all knowledge in the physical sciences is of universals, not particulars. That is, the kind of thing that constitues physical knowledge is a statement of the form 'Such and such occurs whenever the conditions are so and so' and it is the appearance of the word 'whenever' that marks this form as being a universal. The fact of its universality means that we can use such statements combined with imprecise particular statements like 'the table is 1.7 m with an error of less than 0.001' in a deduction, because all we need to ensure the validity of the deduction is that the range of possible measurements is contained in the so and so necessary conditions for such and such to occur. So we can take a universal like 'whenever the table is more than 1.8 m long it will not pass through the door on the stairs' and the indefinite statement concerning the length of the table and deduce that this table will not pass through that door. We do not need the statement about the length of the table to be true for this deduction to be a valid deduction in the sense that the consequence is necessarily true when the premisses are true. That is, we don't need to know the truth values of the premisses to know it to be a valid deduction. The deduction takes the form of a prediction which can be tested, and in the event that the prediction is found to have been false we can investigate what was the reason and this will be found to lie in either one or other or both of the premisses being false. The question is then how are we to know what are the true premisses on which to base our deductions, and the answer cannot be that they are the consequences of deduction because the question would be the same regarding the premisses of whatever deduction we suppose it to be.

## 17 Inductive inference

What we need to explain is how it is that we can come to know a universal in the first place, as a basic truth. And the answer to this is 'by induction'; i.e. we learn a basic universal truth in just the same way we come to know the meanings of the words we use to describe it, because the knowledge is nothing more or less than the interpretation of its verbal representation. The basis for this belief is itself inductive: I believe that human beings are so constituted as to be able to learn to use language when I observe this clearly in, say, my daughter, who happens to be true to type according to everything I know about her and the species we call human being. This does not mean I know everything about human beings, or everything about my daughter. There is surely more to know, but the basis for this further knowledge will always include the knowledge of them as it now stands.

The remarkable fact is that, using deduction on the basis of only vague perceptions of universals and indefinite utterances concerning particulars, we can move from a state of vague knowledge to one of more precise knowledge. To do this we need to choose carefully what we consider to be the basic truths. The basic truths in any deduction must be 'prior and better known to us' than the conclusion. 'Better known to us' because otherwise we would not be making any advance, and 'prior' because we require the basic truths to be the cause and the reason why the conclusion is what it is and not otherwise, because when this is the case, the conclusion could not be other than what it is. Moreover they need to be 'appropriate' to the subject.

## 18 Essential and accidental attributes

There are two senses in which something is said to be better known: one in sense-perception and the other in being. Things better known in sense-perception are better known to man, and those better known in the order of being are less well known to man. For example we can know a particular tree better than we know the species of the tree because we know the tree in immediate sense-perception, about which it is hard to be mistaken, but the species is only known to us indirectly by deductive inference, which is much less certain because the definition of the species may be not be sufficiently precise (for example if the species is actually a sub-species of an unkown species).

The order of being is the form of things in abstract representation: the hierarchy of animal and plant life, to take the most obvious and famous example. This is abstract, because the species are not substantive wholes like an individual animal is—they are abstract classes each within a genus and characterised by predicates, which are abstract forms of utterance. By calling this abstract representation 'the order of being' we acknowledge that it is real and not just an artifact of the way we represent animals and plants as belonging to certain classes. Our knowledge of biology is not a collection of arbitrary facts: it is only knowledge because it can in principle be experienced to be true or false by anyone who understands the language in which it is expressed and who can judge that it accords to the facts of experience.

The progress of knowledge then is from generalities which are vaguely perceived in sense-perception, to universals, which are things better known abstractly than their manifestations are known in sense-perception. In Aristotle's terms, the more abstract things are 'better known in the order of being' than the less abstract: for example 'living thing' is quite simple, being defined as something along the lines 'Either an animal or a plant'. But the less abstract, such as some particular species of plant, are much more difficult to define as witnessed by the complexity of botanic keys which are an attempt to guide one through a process of identifying a particular specimen down to some type. Now 'living thing' is better known in the order of being, but very hard for us know directly: for example we have a range of what are to us quite well-known physical systems, including individual rocks, piles of sand, Rayleigh-Beynard convection cells, autocatalytic chemical reactions, prions, viruses, bacteriophages and then single-celled organisms. To know where the boundary is between the living and the non-living we need to know which of these are animals or plants. We will very likely need to make a finer discrimination between the living and the non-living before we understand the reason life is. Other examples of things in this class are matter, energy, space, time, consciousness and truth. The more abstract things are harder to know through sense-perception, but prior to and better known than the less abstract in the sense that less abstract things are described in terms of the more abstract, so that to understand the less abstract we have to assume the more abstract.

Plant identification is surprisingly difficult to learn but provides a very good example of the movement of knowledge from vague perceptions to precise definitions. Many people cannot identify plants beyond descriptions like 'tall with white flowers'. This is what Aristotle would call 'a vaguely perceived generality' and such vague generalities give rise to inductive inferences like 'tall plants with white flowers always grow in the same place that tall plants with red flowers grow'. Such an inference is quickly discovered to be false and this prompts a search for finer discrimination which if found, results in the vague generality of 'tall white flowers' being seen as a composite of two or more somewhat less vague generalities, all of which are 'tall plants with white flowers', but which have, say, different types of leaves, those of one being made up of lots of little leaves. In doing this we simultaneously refine the vague generality 'leaves' into 'composite leaves' and 'other leaves', perhaps remarking to ourselves that it is odd that we hadn't noticed this very obvious difference before. But having done this we are more likely to notice that a plant, whether a flower or a tree, has composite leaves or not, and we find that in fact many other species can be discriminated by this same feature. This new discriminatory ability thus leads us to know the plants better 'in the order of being'.

Now some equally obvious features of plants are not actually essential characteristics, but what Aristotle calls 'accidental'. These are such as the colour of flowers which in some species can vary considerably depending on the conditions in which the plant grows, or in the case of Mendel's peas, on the particular genetic history of the plant. But the fact that we can discover that these are indeed not essential to the species is what leads us to know the order of being as something real and not just a figment of our imagination. It is only through the essential attributes of the species that we know that the others are not essential. For example by an experiment in which we cross-pollinate each type with the other and then grow some of the seedlings of each in each of the two environments, with pure-bred controls drawn from each type. The conclusion then follows from a positive observation of the second generation sharing the same essential features, yet having colours which are in accord with the place where the plants grow, and not their ancestry. This verification depends on being able to identify all the plants involved as being of the same species.

The fact of essential attributes is what allows us to differentiate between the various abstract things we come to know indirectly through reason. The essential attributes of things are their defining characteristics in the sense that without these attributes the things would not be those things. The word *essence* comes from the first Roman translators of Aristotle and it is a word they invented to carry the meaning of his phrase 'the what it is to be'. So the definitions of the subjects of true knowledge are real, and the discovery of symbolic representations of them is the aim of the scientific method Aristotle describes.

Aristotle's term for absract idea is eidos which is traditionally translated as *species*. Each species (abstract idea) is part of a whole, the generic idea, which is the generalisation of the ideas which fall under it. This is traditionally termed the *genus*. The essence of a species is then defined by giving the genus and a set of attributes called the *differentiæ*, which serve to pick out that particular species (idea) from the genus (generic idea). The hypothesis of the existence of true knowledge is that there is a natural generalisation of the ideas; i.e. they are not arbitrary or contingent upon the order in which we discover them.

True knowledge is the knowledge of the natural definitions of the ideas. Together these definitions are called *the order of being*. And it is the order of being which explains the causes of things, i.e. the real reasons why they are as they appear to sense-perception. In this sense then the genus is better known in the roder of being than the species, because we need to know the genus before we can know the species. But in sense-perception it is the species that is better known. Knowledge is the knowledge of the order of being, and it proceeds in opposition to the kowledge we obtain through sense-perception. The latter proceeds from the vaguely perceived whole (a species) in terms of its better known particulars (the individuals): to the more clearly perceived whole which gives us a complete explanation of the particulars, i.e. of what we see. We then know the particulars in a different sense: as being of that clearly perceived whole, so we know the 'what it is to be' of the particulars, whereas before we knew them only superficially through sense-perception. So we have moved from extensional knowledge to intensional knowledge. Hume's error was that he failed to recognise intensional knowledge as real and he presumed extensional knowledge was the only kind of true knowledge, from which he validly deduced we cannot really know anything.

## 19 Aristotle on Euclid

Aristotle lived in Athens from 335 until 323 BCE and died in 322 BCE. Little is known about the life of Euclid except that he lived in Alexandria, in what is now Egypt, around 300 BCE. It is very unlikely that Aristotle ever saw a manuscript of the *Elements*. Yet all of Aristotle's geometrical examples can be found there so it seems very likely that Euclid was following Aristotle's methodology to organise geometry, and furthermore that he was doing it 'by the book': the whole text of the *Elements* is a showcase of Aristotelian method in the mathematical sciences.

There are some ancient and well-known puzzles about why Euclid structures the definitions and postulates of the Elements in the way he does. Most famously is the problem of the so-called 'parallel postulate'. There is in fact no postulate that deals with parallels. There is a definition of parallel, and a postulate that is often construed to be about parallels but in fact concerns only lines that are not parallel. The famous question was 'Why did Euclid define parallels[27] and not simply construct parallel lines, i.e. co-planar lines which could be proved to have the property of not meeting when both are produced indefinitely in either direction?' The answer is because it is a self-evident fact that pairs of parallel *planes* exist. Given a square (defined as a right-angled equilateral four-sided figure) if you rotate it around one edge you will get two parallel planes described by the edges connected to the axial edge. This, along with the fact that Euclid can construct parallel lines and planes from the other basic definitions provided he has the postulate that he can construct trilaterals.

To see why parallel lines and planes are assumed as self-evident empirical fact we need to understand the definitions in terms of which it is stated. The generic subject of geometry is location. The way to understand the definitions is to recall that geometry is an empirical science and that it aims to describe the gross observable features of our sense-perception as it relates to the locations of things. What we directly experience of our surroundings is one whole we call the space, which is divided into disjoint parts, each of which are themselves wholes in some different sense: these are the things we can see and touch, i.e. the substantial things. So we might describe our surroundings like this: the bottle is near the edge of the table, and the water is in the bottle. The table stands on the floor. There is air in the room. The corner of the table and the lower half of the bottle are in sunlight coming in through the window. The feature that geometric intuition abstracts from this experience is the sense of something's substantive being in relation to the substantive being of other things. These are all objects of our experience: we can see them and touch them and breathe them and as Aristotle says, knowledge proceeeds from these things 'better known to man' to those things 'better known in the order of being'. The vague sense in which our surroundings are one whole is that there is an internal coherence to the description: the bottle is not both on the table and on the floor, for example.

The process of definiton analyses this general and rather vague sense of wholeness of geometric being into it's particular senses. The analysis is in terms of *division*. What concerns us in relation to location is not the material of the things, but their

[27] Parallel lines are defined as pairs of co-planar straight-lines which don't meet when continued indefinitely in either direction and parallel planes are defined as pairs of plane surfaces that don't intersect when extended indefinitely in any direction.

form. So we are analysing the form in respect to location. In the room as a whole, what divides one location from another is always one or more surfaces: it's the surface of the bottle in contact with the surface of the table that divides the location of the bottle from the location of the table. The relations between the surfaces of each separate part is the reason why we say the objects don't have the same locations and ultimately why they are geometrically separate objects. But its surface is not a substantive part of any thing: we could not perceive the surface directly, and it cannot exist apart from the thing.

From these particular observations we conceive by induction the universal which is that, in the geometric sense, the substantive object is the cause of its surface. In geometry then, substantive things are prior to and better known to us than their surfaces. Now we have defined surface and we also have a clearer idea what we mean by the tortured phrase 'the sense in which the object is, considered as something that has a location in space': it is that the object is a geometric *volume* bounded by a geometric *surface*. And we know that all of geometry is going to be statements about the relations between volumes just in terms of which objects share which of their boundaries with which boundaries of which other objects. So it is clear that Euclidean geometry is not the geometry of empty space: Euclid has nothing whatsoever to say about empty space.

Now when we consider some surfaces we see they too have boundaries of a kind because we can discriminate between different parts of a surface. For example the inside of a bottle and the outside, or the top of the table and the sides. The different parts of a surface are not determined by the form of the volume however. Parts of one surface are always differentiated from each other by other surfaces. For example, part of the bottle might be in sunlight and part not, then that part of the outside surface of the bottle will be illuminated differently to the other part. The part of the volume of the room in sunlight thefore has a superficial boundary with the part that is not in sunlight, and this suface meets the surface of the bottle and therefore allows us to discriminate between the two different parts of the latter: that part in direct sunlight and that part that is not. And contrariwise, the transparent surface of the bottle intersecting the boundary surface of the sunlit part of the room divides the latter surface into the part that is inside the bottle and the part that is outside. [28]

We perceive intuitively that definite surfaces cut one another along definite lines. So we conceive by inductive inference the notion of the parts of all geometric *surfaces* being bounded by geometric *lines*, and as we see that surfaces define the geometric forms of volumes, we see also that lines define the forms of surfaces because it is only through being able to distinguish the different parts of the surface that we can determine its overall form, and the parts of surfaces are divided by lines lying in the surface. When we see a surface with three parts $A$, $B$ and $C$, say, with common boundaries we see that one part of the boundary of $A$, say $AB$, meets the boundary of $C$ which then divides into two parts $CA$ and $CB$. So we see that lines discriminate between different parts of a surface the boundary lines of surfaces are divided by the geometric *points* at which they cut one another.

Here the process of division comes to a halt because if we could discriminate between the parts of a point then the boundary $AB$ would meet the boundary of

[28]Geometric topologists should note that it is not by knowing the form of the bottle itself that we know the division between its inside and outside surfaces, it is by knowing the different ways in which the surface of the bottle divides the whole volume of which the bottle is a part. The complete set of topological invariants is going to have to characterise all of this, so it will not be something that can be determined from a symbolic description of just the surface described as an incidence matrix.

$C$ in an indefinite location so the boundary between $AC$ would be indefinite and it is not. When we consider the line where one definite surface cuts another definite surface we see that it is definite in the sense of not having a width that could be divided, because if it had width it would not make a definite division of the surface. Then similarly the sense in which a surface is a definite division between volumes is in the sense that its depth cannot be divided otherwise the surface would not divide the volumes definitely. Finally it is volume that has extension in length, breadth and depth.

We say a surface is uneven when we can discriminate between the different parts by touch. So saying that a surface is even means that one part of the surface is in some particular sense equal to another part, and we need to make this sense precise. Now a spherical surface is an even surface, as is the face of a cube, but the corner of a cube is surely not, and so neither is the whole surface of the cube. Intuitively, the property of being an even surface is that both parts have the same inclination at their common boundary, so we say that an even surface is one which lies evenly with the lines in it. But we must remember that surfaces are not actual, and neither are lines. Whole surfaces are the boundaries between the parts of whole volumes, and lines are the boundaries between parts of whole surfaces. So the lines in a surface are just the intersections of that surface with another surface and they are the result of the whole volume having been divided in two different ways so that it falls into four parts. Thus two intersecting surfaces are even when each lies evenly with the common boundary. Three intersecting even surfaces are plane when each lies evenly with both of the common boundaries.

Now all these considerations were completely general with regard to the exact size, orientation and position of any of the surfaces concerned, and so the defininitions of even and plane surfaces are properties of the whole surface, not just local properties at some particular boundary. So we see intuitively that the properties of even surfaces as we have defned them apply at all points along the lines which represent the intersections of those surfaces. These properties therefore translate along the lines in such a way they are preserved and it is the character of the surfaces that guarantee this.

But these surfaces are just the boundaries between intersecting volumes. So we have a notion of a volume lying evenly with the surfaces in it and thus we have the intuitive idea of there being volumes of even and odd character, which are the irregular and the smooth, and that the smooth volumes are further divided into straight and curved. But we cannot define these because we have no empirical intuitive basis for starting with the supposition that there are hyper-volumes: we have to start with volumes because these are the geometric abstractions of our actual sense-perception of substantive wholes. Nevertheless, we can characterise the volumes as if there were a hyper-volume which determined them. When we do this we can 'see' that of the volumes which we think of as smooth, the curved and the straight would correspond to the evenness of the volumes with respect to their spherical and planar intersections. These properties of the volumes would therefore translate covariantly in the surfaces in the analogous manner to that in which the local evenness and straighness properties of the surfaces translate along linear and circular boundaries. The spherical intersections thus correspond to rotational symmetries and the planar intersections translational ones.

Because lines are just intersections of surfaces this gives us the notion of straight-lines at the same time. A straight-line is just the boundary between two plane surfaces and so a line is straight just when it lies evenly with the points in it. The only kind of even line is the straight-line however: because points have no parts we cannot discriminate between the even and the uneven, so we cannot divide a species of 'straight point' from the generic point.

Thus the fundamental elements of Euclid are points, lines, surfaces and volumes. A point is that location 'of which there is no part', and a line is 'length without breadth'. The extremities of a line are points. And a straight-line is a line which 'lies evenly with the points in it'. A surface is 'length with breadth only' and the extremities of a surface are lines. And a plane surface is a surface that 'lies evenly with the stright-lines in it'. Then a boundary is defined as an extremity of something, and a figure is a surface contained within a boundary, and a rectilinear figure is a surface bounded by straight-lines. A quadrilateral figure is one bounded by four straight-lines, and a square is a right-angled equilateral quadrilateral. A pair of co-planar lines are parallel if they do not meet when either is extended indefinitely in either direction. A volume is length, breadth and depth and the extremities of a volume are surfaces. A pair of planes are parallel if they do not meet when extended indefinitely in any direction.

From this foundation Euclid builds the structure which explains many evident geometric truths, which are sometimes far from evident to the unaided intuition. The entire work is towards proving one astonishing theorem which is that there are exactly five ways in which a volume can be closed off by identical equilateral and equiangular plane figures. These are the platonic solids: the tetrahedron, the cube, the octahedron, the icosahedron and the dodecahedron. When inscribed in the same sphere the first three prove to have sides which are in rational ratios to one another. The remaining two have irrational sides with respect to the diameter of the sphere: they are the irrational called the minor, and another irrational called the apotome.

Finally Euclid proves an un-numbered lemma: that the interior angles of an equilateral equiangular pentagon are one and one fifth of a right angle. This was used in the proof that there are no other possible equilateral figures which can enclose a volume. The significance of this is that the pentagon is the third geometric element. Just as the numbers are the monads, the dyads and the triads, then the rest built up from these. In geometry the elements are the triangle the square and the pentagon: two right-angles, four right-angles and six right-angles. The unit is therefore two right-angles: a triangle, the dyad is two units: a square, and the triad is three equilateral equiangular pentagons which contain a solid angle whose sum is three and three/fifths of a right angle.

There are two ways to arrive at a definition. Plato knew of only one of them, called *division*. The idea is to determine the *differentiæ*, which are the predicates that pick out a particular species from the genus. They must be universal in that they hold of all members of that species, primary in that they hold primarily of the species, and not some higher class to which the species belongs. They must also be *commensurate* which is something best explained by example. We can differentiate three types of triangle as acute-angled, right-angled or obtuse-angled. We can also differentiate triangles by the lengths of their sides, as equilateral, isosceles or scalene, according to whether their sides are respetively one, two or three different lengths. These two differentia are not commensurate however: we cannot use them both to identify triangles as we would have to put one or other as primary and it is not clear which it should be. To be a useful differentia of a species, an attribute must inhere essentially in that species as a whole, so that it can be considered as a cause or a reason why any member of that species is indeed a member of that species and no other.

Why not define triangle as 'trilateral figure' which is the way many people presume it to be defined? Because that would not tell you anything about a triangle except that it has three sides. You would not be able to prove anything about triangles at all from that. Contrast this with the case for quadrilaterals: here there are characterisations of quadliaterals by their angles and the lengths of their sides, but they are commensurate in that the two different classifications don't overlap. Thus in Euclid's *Elements*, *triangle* itself does not appear as a definition: there are just the six peculiar types of triangle, specified separately as the two classes. So any particular triangle is, by definition, both exactly one of the first class, and exactly one of the second class, but it is a matter of demonstration which particular combinations are possible and which aren't.

The definition of *triangle* as a species within the trilateral figures is not by division, it is given by *proof* in the form of Proposition i.32 *viz.* 'If in any triangle one side is produced then the exterior angle is equal to the sum of the two opposite interior angles and the sum of the internal angles is equal to two right-angles.' This is the true differentia of triangle as a species of trilateral. The quadrilaterals are defined by division, because there the two classifications by angles and lengths of sides are commensurate, so the divisions 'match up' and the quadrilaterals can be divided into square (right-angled equilateral) rectangle (right-angled not-equilateral) rhombus (not-right-angled equilateral) rhomboid (not-right-angled not-equilateral opposite sides and opposite angles equal) and trapezoid (not-right-angled not-equilateral not rhombus). This would not work in the case the case of the species triangle because we would have to prove the differentia are universal: it is not obvious from the two separate classifications by angles and sides that this is in fact all trilateral figures, which is what we would need in order to know anything about them: a universal stating 'All triangles . . . ', but this is what we do get from Proposition i.32. It is also clearly primary because it applies to just triangles and no other rectilinear figure is a triangle.

As well as knowing the definition of something, one needs to be able to show that it exists. Having proved that all triangles have angles equal to two right-angles, we need to be able to construct them. Thus when Euclid writes 'let ABC be any triangle' he makes implicit reference to Postulate i.5, the so-called parallel postulate, to assert that the thing defined can actually be constructed. This postulate is not about parallel lines, it is about the existence of trilateral plane figures. But it does have an indirect connection with parallels via Definition i.23 of parallel lines as 'straight lines which, being in the same plane and each produced to infinity in both directions, meet with one another in neither.'

Given any three distinct coplanar straight lines produced to infinity, it seems that there are only four possible results: they intersect at none, one, two or three points. So the result is either three parallel lines, two parallel lines and a third which falls across them because it is parallel to neither of them, or a plane figure, bounded by the three straight lines which could possibly be a 'degenerate' trilateral consisting of just one point. These can all be constructed by combining postulates or propositions trivially. (i) The case of zero points of intersection is Proposition i.30 'Straight lines parallel to the same straight line are parallel to each other' applied to two applications of the construction of Proposition i.31. (ii) That of one point of intersection is a trivial consequence of Postulates i.1 and i.2. (iii) That of two points is by Proposition i.31 and finally (iv) that of three points is by Postulate i.5. So the logical consequences of having three distinct lines each produced to infinity in both directions fall into four parts: the 'trivial' cases of zero or one point of intersection and the non-trivial cases of two parallel lines with a third falling across them, or that of trilaterals. Only the last of these is a plane figure..

The definition of parallel is merely the definition of the meaning of the word. The question Aristotle considers is whether in fact Definition 23 could not be replaced by a demonstration, i.e. by providing an explicit construction which, given one straight line $AB$, constructs another straight line $CD$ such that when both lines are produced continuously in either direction they meet in neither. In other words, something like the construction produced by Proposition i.31, except not referring to the definition of parallel, but just proving that the two lines when produced to infinity do not meet. And his answer is quite straight-forwardly 'no', and this is simply because there is no other definition or postulate that mentions two straight lines *not meeting* when produced to infinity. A proof by contradiction is indeed possible, and this is what Euclid does in Proposition i.27, however this proposition concerns pairs of (coplanar) lines which have a third falling across them and which make 'alternate angles equal.' It then proves by contradiction that the first two lines do not meet when produced to infinity, and that they are therefore parallel. The construction in Proposition i.31 produces exactly this condition of equal alternate angles, and then invokes i.27 to prove that they are parallel lines.

This distinction between the knowledge of the meaning of the word and the knowledge of the fact is very important. Aristotle uses the definition of triangle to illustrate the distinction in two different ways. First he uses it as an example of something that is known as being the meaning of the word: i.e. that triangle means just 'any figure whose angles are equal to two right angles.' And then later in the same paragraph he uses it as an instance where judging something as having been demonstrated depends upon previous knowledge as well as knowledge obtained simultaneously with the knowledge of the fact:

> Recognition of a truth may in some cases contain as factors both previous knowledge and also knowledge acquired simultaneously with that recognition-knowledge, this latter, of the particulars actually falling under the universal and therein already virtually known. For example, the student knew beforehand that the angles of every triangle are equal to two right angles; but it was only at the actual moment at which he was being led on to recognize this as true in the instance before him that he came to know 'this figure inscribed in the semicircle' to be a triangle.

This too is an example that we can find in Euclid: it refers to Proposition iii.31 which states 'In a circle, the angle in a semi-circle is a right-angle, and that in a greater segment less than a right-angle, and that in a lesser segment greater than a right- angle. ...' Here the objects of discourse are 'angles in segments' which, though they are particular angles of triangles, are defined completely separately from triangles (Definition iii.8 'And the angle in a segment is the angle contained by the joined straight-lines, when any point is taken on the circumference of a segment, and straight-lines are joined from it to the ends of the straight-line which is the base of the segment.' But during the course of the proof, it transpires that the angle $ABC$ in a segment together with the angles $CBA$ and $BCA$ is equal to two right angles. At this point Euclid writes that $ABC$ is a *triangle*. The proof that this triangle has its angles equal to two right-angles is quite different to that of Proposition i.32, so though the student was presumably familiar with the latter, he discovers that the angle in a segment is in fact a triangle, and this is according to the definition of a triangle as just a word which is short for 'a figure whose internal angles add up to two right-angles,' the fact of which was previously known to him from the proof of Proposition i.32, but for a different reason, because the proof in iii.31 is only for angles in a semi-circle which happen to all be right-angled triangles. So it is only when he sees that this particular angle in a segment has internal angles equal to two right-angles that he recognises it as a triangle.

## 20 Truth revisited

The Truth is not the same as the collection of true propositions, because as we saw, propositions are just the forms of certain types of utterances, and so the class of true propositions is undefined: two utterances of the same proposition may have different truth values, having been uttered in different contexts and only when the two contexts are integral parts of one whole can we put the two propositions together and decide which is true and which is false. Neither can Truth be the collection of true utterances because these are each only judged true according to the truth of the propositions and these utterances may contradict one another because some of the judgements may be mistaken. But it is precisely because we can know when we have made a mistake that we know the Truth is independent of our particular beliefs, however unshakeable the foundation of those beliefs.

This unshakable conviction in our scientific beliefs is necessary because the only way we come to know of errors in our thought is through contradiction. Contradiction is a sure sign of erroneous knowledge because all knowledge is only considered to be such when it can be expressed effectively in words and compared with experience. But a contradiction cannot be compared with definite experience because it is by nature indefinite: it states that something is at once so-and-so, and yet not so-and-so. So we need to be certain of our beliefs because only then can we be certain of the contradictions. And this is what we all experience: that people who think clearly and are certain of their knowledge are able to know much better when there is somethng they don't understand because they run into a definite contradiction. They can then immediately set about finding out the right way to resolve it. Those who do not have this conviction in their beliefs are not sure when they see what appears to be a contradiction that it is indeed a contradiction. It may just be an erroneous observation. So they are not so inclined to look for an error in their thinking. But this state perpetuates itself, only it increases in uncertainty at every step. In its extreme form we have quantum physicists who are trying to explain quantum mechanics but begin to doubt that logic is right, so they invent quantum logics. In doing this they immediately introduce the uncertainty of their own reasoning into their experimental evidence which is the ultimate basis for everything that they believe about quantum mechanics. Aristotle writes that those scientists who question what is admissable as true or false 'suffer from want of training in logic, for they should know these things before coming to hear lectures on their subject'. The result of such reasoning is that things are as they are because that is how they are, which as Aristotle points out elsewhere is 'an easy enough way to explain anything.'

This dependence of Truth on intuition means one cannot distance oneself from one's thought: intuition is always personal. There is no sure way to be right; we cannot devolve the whole of our reasoning processes to a calculus because we must always first decide what are the exioms it should use and these must always be based on intuition. On the other hand what we genuinely know at any one time *is* how things actually are: those things we do not know are completely obscure. We only truly learn when we discover something that is genuinely new, which was not merely a hitherto un-noticed consequence of those things we already knew before, which would amount to just a different way of expressing the same actual state of affairs. So we can only learn if we are prepared to change our minds about how things actually are. Therefore to apprehend Truth one's intellectual integrity must be greater than the integrity of one's ego. It is always a personal matter: one must stand behind one's judgements and be prepared to suffer humiliating defeat if one is found to be in error.

Perhaps it is better then to be humble from the beginning, but this does not mean prefacing everything one says with 'unless I'm mistaken' because there is no basis for reasoning from indefinite knowledge. The humility needed is that of an ice climber. An ice climber has to be certain that the protection they have put in will hold a fall, so they use multiple backups because they know that any one could fail. Their certainty is based on their intuitive judgement about the probabilities of the different points failing and the probable consequences. The certainty is based on their knowledge of the possibility of failures of individual points, so failure is always a possibility. But each point of protection is a back-up for the others. The climber thereby stakes their life on their certainty that they are protected from a bad fall, but at the same time they know that it is possible that they will die. The contradiction is only apparent, because certainty is not necessity. If two people were to repeatedly flip coins, it is certain that eventually one will come up heads and the other tails. This is not necessary though. So it is possible that they continue flipping the same result indefinitely, but it is certain that they won't.

Philosophy is the opposite of ice climbing. Wisdom is knowledge realised in multiple interdependent systems of thought in such a way that if one fails the others must all fail too. So the basis of wisdom is necessary truth in the sciences. The certainty of the philosopher is that true knowledge is a necessary consequence of the necessary truths in the sciences. Aristotle explains the difference between those with wisdom and those with particular knowledge by analogy between an architect and a builder. The builder knows the material cause of the building and he is himself the efficient cause in that he knows how to arrange the matter into the form that makes it a building. The architect does not necessarily know this, but he knows the reason why the builder must do what he does to make the building. In other words the bricks and mortar are the material cause, the builder's knowledge of how to arrange the material into the form of the house is the efficient cause and the architect's plan is the formal cause. The builder's knowledge of the house is complete: he knows the efficient cause, the material cause and the formal cause and he can build as many houses as he likes. But they will all be the same form. The builder lacks the architect's knowledge which is the knowledge of the *reasons why* the house has that particular form. The architect's knowledge is second-order knowledge about knowledge and as such it is the final cause of the house.

It seems likely to me that Freemasonry originated in this analogy of Aristotle's. A mason was traditionally first an apprentice and then after many years working under a master, he himself bacame a master mason and had serving apprentices under him, from whence come the terms under-standing and under-study. But the master remains a mason. The free mason is a mason who has come to knowledge of why it is that he does what he does, so he has transcended the knowledge of his craft and become the architect. The remarkable imagery on the reverse of the United States one dollar bill is inspired by freemasonry. Franklin D. Roosevelt is said to have been behind the appearence on the currency in 1935. Roosevelt was impressed by the masonic imagery when he noticed it on the reverse of the Great Seal of the United States which is the seal used by Congress to approve changes in the federal law. The image is of a truncated pyramid, with just the lower thirteen rows of bricks. The apex of the pyramid is completed by a two-dimensional triangle in a radiant blaze, through which gazes a single human eye.

The thirteenth proposition of the thirteenth book of Euclid is the construction of a pyramid. The construction is of one particular form of what is generically defined as being a solid figure contained by planes, constructed 'from one plane to one point'. The number and angles of the planes and the lengths of the sides are indefinite and the construction works for any number of planes. The particular construction of Proposition XIII.13 is a pyramid with three sides and a triangular base, but the construction generalises to any number of sides greater than this. It is not just a construction, but a proof as well: it is to construct a pyramid in a given sphere and then to 'take it around in the sphere' and thereby show that the square on the diameter of the sphere is one and a half times that on the side of the pyramid.[29] The remaining propositions in Book 13 are the constructions and characterizations of the squares on the sides of the other platonic solids, followed by Proposition 18, which is the *denouement* of the whole work: the packing of all five Platonic solids into a single sphere and the proof that there are no others. The demonstrations of these propositions are long and difficult. They require all aspects of the other books, especially the theory of the incommensurate proportion.

So the significance of the truncation of the pyramid seems to be that after proposition thirteen, the remainder of geometrical knowledge is transcendent. It is presumably left to the mason to wonder what is the use of proposition thirteen. Only when he understands Proposition 18 can he see that the construction in Proposition 13 is generic, and that the particular species of pyramid that motivates it is the tetrahedron which is one where all the sides of the figure are equal so that the pyramid has three sides and a triangular base and all the triangles are equilateral.

The pyramid on the reverse side of the Great Seal of the United States is a four-sided pyramid with a square base and the sides are clearly not equilateral. However the very first design for the Great Seal produced by William Barton of Philadelphia is ambiguous: only one face of the pyramid shows, but it is very nearly equilateral, as is the design he copied it from, which was done by Francis Hopkinson in 1778 for the $50 bill of the continental currency. The final design produced by Charles Thomson has the added triangle around the eye and the pyramid is clearly not a tetrahedron because we can see two sides from which it is clear that the pyramid has a square base.

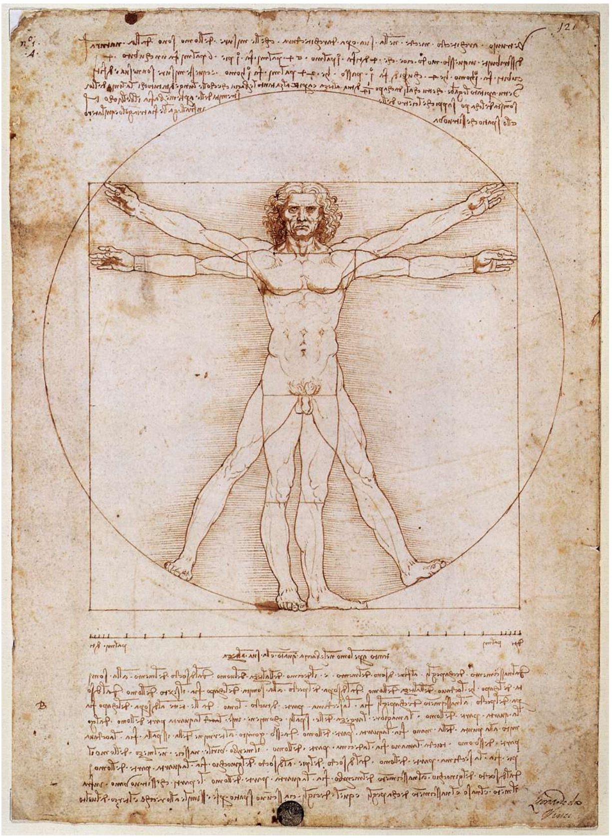
The central theme of Aristotle's philosophy is opposites, and the central opposition is that between the one and the many. Aristotle was constantly arguing against Parmenides, Heraclitus and Plato that all was not in reality one, and that the many are real. Aristotle was arguing for the reality of the everyday objects of sense-perception, and at the same time for the unity of the understanding that it would be possible to have about the causes of these myriad things. His explanation hinges on the opposition between one and many which is the abstract character of our sense perception: we perceive wholes and in this act there is a division between the perceiver and the perceived. Understanding the true nature of reality is a matter of understanding this division in the different ways it manifests itself in our perception. This is how the true and the false are characterised; it is also how the equal and the unequal and the odd and the even are characterised, in all their different senses as these words are used in logic, arithmetic, geometry, harmonics, stonework and poetry. Aristotle said that

[29] The side being any of those lines from the given point to the base.

movement is always from the more perfect towards the less perfect and this seems to be the case with human knowledge in general and the knowledge of the freemasons in particular. It does not seem likely that Roosevelt actually understood the meaning of the symbols he had put on what is possibly one of the most widely circulated banknotes in history.

The reward for our humility in the face of the truth is just a glimpse of how deeply misguided we can be whilst at the time certain in our beliefs. To take Aristotle's example: from the density property of rationals, we are intuitively convinced of the inexhaustible expressivity of the rational numbers as regards ratios of geometric lengths. It is inconceivable to many that there could be a definite length which cannot be measured by any definite unit of measurement whatsoever. Now an obvious ratio is that between the length of the diagonal and the length of one side of a square, and this seems that it should be an easy enough thing to construct. So it is at first considered bizarrely counter-intuitive and apparently inexplicable that, although we may divide any rational number into smaller parts indefinitely, we can never find a rational whose square is two. Yet once we understand the proof of this fact—i.e. the reason why it is so—we are 'in the contrary and better state', because in fact nothing would be more surprising than if we *could* do this: for then odd numbers would be even numbers, one sphere would be two, dead cats would also be live cats and there would be mathematical theorems that were neither true nor false. The defining characteristic of Truth is that it 'conforms to itself in every way', so Truth is only remarkable when it is not properly understood.

But the fact that we can discover this to be a fact: that human knowledge proceded from the more prefect to the less perfect as it was handed down from Aristotle is proof that true knowlege is possible, and that whilst our particular knowledge does indeed proceed from better to worse, the wisdom in the world proceeds towards 'those things better known in the order of being'. In other words particular human knowledge starts with wisdom and procedes the way of senescence, towards corruption and death. But it is in this very process that human wisdom as a whole proceeds against the flow, moving from what is better known to man, to what is better known in the order of being. This is the how the system works: the merely qualified approval of the one, as in the sense of the third person singular, produces the new order of the genera. One is the direction of mortality, the other the direction of divinity. God is a creation of the human mind in its own image, and it is towards this image that the human mind as a whole moves: from the perishable towards the eternal. And this was the basis of Aristotle's moral philosophy: man is a rational animal and he is happiest when he reasons excellently. Aristotle called philosophy the divine science. Wisdom is the good of man, and the final cause of everything we do. The architect is man himself as a whole species, the divine proportion.



# 21 Appendices

## A — A Logical Paradox

In an article in *Mind*[1], Lewis Carroll describes a problem with the interpretation of hypothetical deductions in symbolic logic. The problem is that if logical consequence is to be a reflection of the actual reasons why a statement is true then it needs to distinguish between symbolically identical antecedents in order not to draw false conclusions by contradiction.

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'What, *nothing* to do?' said Uncle Jim. 'Then come along with me down to Allen's. And you can just take a turn while I get myself shaved.'

'All right,' said Uncle Joe. 'And the Cub had better come too, I suppose?'

The 'Cub' was *me*, as the reader will perhaps have guessed for himself. I'm turned *fifteen*—more than three months ago; but there's no sort of use in mentioning that to Uncle Joe; he'd only say 'go to your cubbicle, little boy!' or 'Then I suppose you can do cubbic equations?' or some equally vile pun. He asked me yesterday to give him an instance of a Proposition in A. And I said 'All uncles make vile puns'. And I don't think he liked it. However, that's neither here nor there. I was glad enough to go. I *do* love hearing those uncles of mine 'chop logic,' as they call it; and they're desperate hands at it, *I* can tell you!

'That is not a logical inference from my remark,' said Uncle Jim.

'Never said it was,' said Uncle Joe: 'it's a Reductio ad Absurdum'.

'An *Illicit Process of the Minor!*' chuckled Uncle Jim.

That's the sort of way they always go on, whenever *I'm* with them. As if there was any fun in calling me a Minor!

After a bit, Uncle Jim began again, just as we came in sight of the barber's. 'I only hope *Carr* will be at home,' he said. 'Brown's so clumsy. And Allen's hand has been shaky ever since he had that fever.'

'Carr's *certain* to be in,' said Uncle Joe.

'I'll bet you sixpence he *isn't!*' said I.

'Keep your bets for your betters,' said Uncle Joe. 'I mean'—he hurried on, seeing by the grin on my face what a slip he'd made—'I mean that I can *prove* it, logically. It isn't a matter of *chance*.'

'Prove it *logically!*' sneered Uncle Jim. 'Fire away, then! I defy you to do it!'

'For the sake of argument,' Uncle Joe began, 'let us assume Carr to be *out*. And let us see what that assumption would lead to. I'm going to do this by Reductio ad Absurdum.'

'Of course you are!' growled Uncle Jim. 'Never knew any argument of *yours* that didn't end in some absurdity or other!'

'Unprovoked by your unmanly taunts,' said Uncle Joe in a lofty tone, 'I proceed. Carr being out, you will grant that, if Allen is also out, *Brown* must be at home?'

'What's the good of *his* being at home?' said Uncle Jim. 'I don't want *Brown* to shave me! He's too clumsy.'

'Patience is one of those inestimable qualities—' Uncle Joe was beginning; but Uncle Jim cut him off short.

'*Argue*!' he said. 'Don't *moralise!*'

'Well, but do you grant it?' Uncle Joe persisted. '*Do* you grant me that, if Carr is out, it follows that if Allen is out Brown *must* be in?'

'Of course he must,' said Uncle Jim; 'or there'd be nobody in the shop.'

'We see, then, that the absence of Carr brings into play a certain Hypothetical, whose protasis is "Allen is out," and whose apodosis is "Brown is in". And we see that, so long as Carr remains out, this Hypothetical remains in force?'

'Well, suppose it does. What then?' said Uncle Jim.

'You will also grant me that the truth of a Hypothetical—I mean its *validity* as a logical sequence—does not in the least depend on its protasis being actually *true*, nor even on its being *possible*. The Hypothetical, "If you were to run from here to London in five minutes you would surprise people," remains true as a *sequence*, whether you can do it or not.'

'I *ca'n't* do it,' said Uncle Jim.

'We have now to consider *another* Hypothetical. What was that you told me yesterday about Allen?'

'I told you,' said Uncle Jim, 'that ever since he had that fever he's been so nervous about going out alone, he always takes Brown with him.'

'Just so,' said Uncle Joe. 'Then the Hypothetical, "if Allen is out Brown is out" is *always* in force, isn't it?'

'I suppose so,' said Uncle Jim. (He seemed to be getting a little nervous, himself, now.)

'Then if Carr is out, we have *two* Hypotheticals, "if Allen is out Brown is *in*" and "If Allen is out Brown is *out*," in force at once. And two *incompatible* Hypotheticals, mark you! They ca'n't *possibly* be true together!'

'*Ca'n't* they?' said Uncle Jim.

'How *can* they?' said Uncle Joe. 'How *can* one and the same protasis prove two contradictory apodoses? You grant that the two apodoses, "Brown is *in*" and "Brown is *out*," *are* contradictory, I suppose?'

'Yes, I grant *that*,' said Uncle Jim.

'Then I may sum up,' said Uncle Joe. 'If Carr is out, these two Hypotheticals are true together. And we know that they *cannot* be true together. Which is absurd. Therefore Carr *cannot* be out. There's a nice Reductio ad Absurdum for you!'

Uncle Jim looked thoroughly puzzled: but after a bit he plucked up courage, and began again. 'I don't feel at all clear about that *incompatibility*. Why shouldn't those two Hypotheticals be true together? It seems clear to me that would simply prove "*Allen* is in". Of course it's clear that the apodoses of those two Hypotheticals are incompatible—"Brown is in" and "Brown is out". But why shouldn't we put it like this? If Allen is out Brown is *out*. If Carr and Allen are *both* out, Brown is *in*. Which is absurd. Therefore Carr and Allen ca'n't be *both* of them out. But, so long as Allen is *in*, I don't see what's to hinder Carr from going *out*."

'My dear, but most illogical, brother!' said Uncle Joe. (Whenever Uncle Joe begins to 'dear' you, you may make pretty sure he's got you in a cleft stick!) 'Don't you see that you are wrongly dividing the protasis and the apodosis of the Hypothetical? Its protasis is simply "Carr is out"; and its apodosis is a sort of sub-Hypothetical, "If Allen is out, Brown is *in*". And a most absurd apodosis it is, being hopelessly incompatible with that other Hypothetical that we know is *always* true, "If Allen is out, Brown is *out*". And it's simply the assumption "Carr is out" that has caused this absurdity. So there's only *one* possible conclusion. *Carr is in!*'

How long this argument might have lasted, I haven't the least idea. I believe *either* of them could argue for six hours at a stretch. But, just at this moment, we arrived at the barber's shop; and, on going inside, we found—

**Note.**

The paradox, of which the forgoing paper is an ornamental presentation, is, I have reason to believe, a very real difficulty in the Theory of Hypotheticals. The disputed point has been for some time under discussion by several practised logicians, to whom I have submitted it; and the various and conflicting opinions, which my correspondence with them has elicited, convince me that the subject needs further consideration, in order that logical teachers and writers may come to some agreement as to what Hypotheticals are, and how they ought to be treated.

The original dispute, which arose, more than a year ago, between two students of Logic, may be symbolically represented as follows:—

There are two Propositions, A and B.

It is given that

1. If C is true, then, if A is true, B is not true;
2. If A is true, B is true.

The question is, can C be true?

The reader will see that if, in these two Propositions, we replace the letters A, B, C by the names Allen, Brown, Carr, and the words 'true' and 'not true' by the words 'out' and 'in' we get

1. If Carr is out, then, if Allen is out, Brown is in;
2. If Allen is out, Brown is out.

These are the very two Propositions on which 'Uncle Joe' builds his argument. Several very interesting questions suggest themselves in connexion with this point, such as

- Can a Hypothetical, whose protasis is false, be regarded as legitimate?
- Are two Hypotheticals, of the forms 'If A then B' and 'If A then not-B,' compatible?
- What difference in meaning, if any, exists between the following Propositions?
    1. A, B, C, cannot be all true at once;
    2. If C and A are true, B is not true;
    3. If C is true, then, if A is true, B is not true;
    4. If A is true, then, if C is true, B is not true.

The following concrete form of the paradox has just been sent me, and may perhaps, as embodying necessary truth, throw fresh light on the question.

Let there be three lines, KL, LM, MN, forming, at L and M, equal acute angles on the same side of LM.

Let 'A' mean 'The points K and N coincide, so that the three lines form a triangle'.

Let 'B' mean 'The triangle has equal base-angles'.

Let 'C' mean 'The lines KL and MN are unequal'.

Then we have

1. If C is true, then, if A is true, B is not true.

2. If A is true, B is true.

The second of these Propositions needs no proof; and the first is proved in Euc., i, 6, though of course it may be questioned whether it fairly represents Euclid's meaning.

I greatly hope that some of the readers of Mind who take an interest in logic will assist in clearing up these curious difficulties.

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As far as symbolic logic is concerned the only solution to this problem is to abandon the idea of logical consequence being a reflection of reality. Indeed this is what modern propositional logic does. Hypotheticals are replaced by material implication and this has the property that if the antecedent is false then the hypothetical as a whole is true, even if the consequent would result in a contradiction. Thus the propositional formula $(C \to (A \to \neg B)) \wedge (A \to B) \wedge C$ is satisfiable if $C$ is true: $A$ is false and $B$ is not determined. Propositional logic cannot discriminate between the circumstances which imply that $B$ is false and the quite different ones which imply that $B$ is true. Thus we cannot reason that, since they are mutually exclusive, $B$ will always be definitely true or false and that there is no contradiction.

Carroll surely knew very well what the problem was because Aristotle is explicit that 'hypothetical deductions are not made through the figures', which is to say hypotheticals are never the subject of evident judgement: we cannot make an evident judgement of a hypothetical precisely because at least one of the antecedents is not actual but hypothetical.

In general $\neg A \wedge (A \to B)$ is a tautology, and is true even if $B$ is replaced by $A \wedge \neg A$. Boolean propositional logic has a deduction theorem: we can prove that $A \vdash B$ if and only if $A \to B$, so the relation of logical consequence $\vdash$ has this property too. Another property of Boolean propositional logic is that all judgements are evident because it has an interpretation in a model of truth assigment: i.e. to make an evident judgement of a statement in propositional logic one need only inspect the associated truth table.

The introduction appears to be some esoteric reference to Euclidean Geometry. Possibly relevant facts are that Carroll's book *Euclid and His Modern Rivals* was published in 1879, fifteen years before this article appeared. The reference could also be to the *Elements* including two 'apocryphal' books 14 and 15 which respectively deal with the ratios between the areas and volumes of the icosahedron and the dodecahedron, and with counting the edges and the number of solid angles and measuring the angles between the faces that meet at an edge in the Platonic solids. Or it could be a reference to proposition 15 of Euclid Book XIII which concerns the construction of a cube in a given sphere and the demonstration that the square on the diameter of the sphere is three times that on the side of the cube. Indeed, the reference to *elicit process of the minor*,[30] the awful puns of 'cub' with 'cube' and the two slightly contrived uses of the turning metaphor, both with respect to time, seem to me to point to the Platonic solids. In the *Elements* the ratio of the diameter of the sphere to the length of the sides of the icosahedron is 'that irrational called the minor', and he tells us that the 'Cub' is happy to lay on a bet as to the fact that Carr won't be in when they get to the Barber's shop.

If I were a better I would put my money on Carroll's having discovered that a

[30]This is the name given to the fallacy of arguing a universal from the similarity of appearances. This is 'in the third figure' and argues 'all $b$ are $a$, all $c$ are $a$; therefore all $b$ are $c$'.

great deal of what he wrote in Euclid and His Modern Rivals was wrong.[31] Most importantly, the basis for his argument for keeping to Euclid's structure should have been made from the internal coherence of the whole theory, and its coherence with the structure of logic itself. Unfortunately at the time he wrote, this would have been a very difficult argument to support because the theory did not actually have any obvious coherence. The problem was that the texts of the time had been badly corrupted, even before the arrival of the modern rivals Dodgson wrote about. Three years after Dodgson's book appeared the first modern Greek edition of the Elements appeared. Until that time all of the texts had come from Arabic translations with very tortuos genealogy. The Heiberg edition was the first to have been translated directly from the early Greek texts. Heiberg's edition was in Latin, and appeared between 1883 and 1885. Dodgson would have been a very eager reader and would have had a decade to ponder over it before writing this.

My guess, and it is just a guess, is that Dodgson found the connection between the formal structure of the Aristotelian syllogism, Euclidean Geometry and the theory of probability and that this explains the connection between incommensurable proportions, the direction of the flow of time, and chiral symmetry in three dimensional space.[32]

## 21.1 B—The Lyceum project

There is a global conspiracy to cover up true knowledge. Though it seems to be extremely effective and efficient, it is not an organised conspiracy; it is completely anarchic. It is just a result of a natural tendency for particular knowledge to become corrupted by people who don't know, but think they do know and are driven to publish this fact by a desire for fame, fortune or perhaps just to scratch a living in academia or as a writer. These people introduce errors into the texts and spread disinformation about it.

A sophist is someone who profits from imparting what appears to be knowledge. The philosopher does not look for true knowledge in books, he looks for it in his own intution. If he finds it there then he can draw knowledge from the most extraordinarily corrupt texts. But without intuition all texts are just empty symbols. The knowledge the sophists peddle appears to be true knowledge, but it can't be because true knowledge is not saleable: you cannot make a profit from selling people something they have to find in themselves.

It is possible that the existence of the Internet has brought about a reversal and that the corruption in human knowledge is no longer inevitable. In Aristotle's day, knowledge worth distributing was knowledge worth copying out by hand, and this was something the student had to do himself: it could not be trusted to a slave unless the work was meticulously checked and this takes almost as long to do as it would to write it out in the first place. Since the student will be reading and studying the text anyway, he would better profit from copying it out in his own hand. So the only knowledge widely distributed was of the very highest quality which motivated the students to copy the texts. One obvious mechanism for the corruption was the invention of the printing press. Suddenly enormous amounts of knowledge appeared in books and these were much more easily obtainable than hand-made copies from a good source. Of course all the best knowledge of the time immediately appeared in

[31] What makes you think I didn't know it was wrong when I wrote it? But yes, there were some things I would have written differently had I seen the Heiberg edition. *CLD*

[32] As it happens, this is correct. But can you *prove* it? *CLD*

print so very soon it became assumed that the only good knowledge was that which appeared in a published book. After this, knowledge had to be something someone could profit from in order to be worth distributing widely. So all knowledge became sophistical knowledge.

The Internet reverses this because, although it doesn't guarantee any knowledge will be spread widely, it at least makes this a possibility, and that is all that is needed. Encouraging signs are that the free software movement is able to create software that is monotonically improving in quality, and it is principally through this work being done for its own sake: the pursuit of excellence in the practical application of reason to the production of software and information. Perhaps then the Internet is the crucial thing that enables the mind of man to act as one whole with a purpose, without being riven by conflicting individual interests.

So I want to start a little Internet project called the *Lyceum*, named after Aristotle's school in Athens which had perhaps the first great library in antiquity. The idea is to collect and make available the best possible original texts, starting with those of Aristotle, and tracing this forwards to his intellectual descendents, and back to his predecessors. The idea is then to abstract from these texts a formal structure called a *model* which explains the internal coherence of the texts as well as possible. The formal model will initially be little more than a set of cross-references. Aristotle uses his logic and philosophy in many branches of science so there are examples spread throught his *ouevre* and these could usefully be linked. The following is an outline design for the system. At this stage the project does not need any transcendent knowledge, just solid masonry. We need master masons to help build a foundational infrastructure: document editors, translators, formatters etc. Then we can begin collecting and linking together the many digital sources of ancient wisdom.

There should not be any central editorial control, the system should be completely open and distributed. The basic framework will be just a definition of a minimal text document format and a system of cross-referencing. This system must be robust, so it needs to track genealogy of texts and sources, and nothing must disappear: old versions should remain available so that links are not broken. It must be possible for different sites to mirror data, possibly on an ad-hoc basis. So when a document is referenced, the reference will include the address of the particular mirror and the version of the actual text the mirror supplied (as a cryptographic checksum) as well as the address of the original document. Each document will therefore have a *posterity* as well as an *ancestry* and the posterity should be updated according to references made from other documents. The posterity of one node is just all those other nodes in whose ancestry that node appears. To verify identity of authorship, documents could optionally be digitally signed by their authors who make their public keys available.

Because of the update-only concurrency model, the system need not make any explicit record of time in document metadata. Everything relevant to the partial order of events will be inherent in the ancestry of the node. If any node 'breaks the rules' by changing or deleting a text rather than adding a new version then the distributed ancestry will contain evidence of this fact. If it does not then there is no evidence that the change ever occured. The principle is that all important parts of texts have public references to them.

The top-level categories will be Quality, Quantity, Relation, Posession, State, Place, Time, Action and Affection. Each generic name will be divided into specific names. The generic name Posession will have a specific name Posession.Name which contains a list Posession.Name.* of first characters of names, each of which contains a list of second characters etc. These nodes are each the numbers which we presume will be of Unicode points. The top-level nodes from Quality onwards are understood to be coded in order 1, 2, 3, 4, 5, 6, 7, 8, 9. These are to be interpreted according to sub-nodes Posession.Name.*, for example Posession.Name.4.Posession.1.Name or Posession.Name.2.Quantity.1.Discrete or Posession.Name.2.Quantity.2.Continuous. So the generic idea of Number for example would be coded as 2.1.1 perhaps, and letters being 2.1.2 etc.

The category Time.* contains the numeric checksum of each version of the toplevel node, and within these, checksums of the subordinate nodes. The only actual thing that appears in any genus is one of these checksums. Since there are never more than these first 9 numbers in any genus we simply use the checksum which concatenates them into a decimal number. The top-level node of a Lyceum system is always 12345789 and all the checksums begin Time.123456789.* Thus there is no data in the system, apart from the temporal order of the interpretation of the top-level genera.

This highly recursive structure may seem inefficient, but—once the fixedpoint has been reached—it has the advantage of generality: everything being interpretable into any natural language whatsoever, including the top-level categories. And the interpretation can be changed without changing the top-level. As long as the Posession.Name category has its own name then the assumption of UTF8 character encoding will produce whatever data has been classified. Even this much is only strictly necessary, because the character set encoding and language could probably be guessed from the name of the Posession.Name category. It also has the property of being asymptotically optimal compression of the data: all common substrings of the data are shared, even with documents that have not yet been written.

Clearly the fixedpoint cannot not be reached from the basis of any concrete foundation. The entries in Posession.Name must therefore be abstract. Here is an example of how this works. We will use the notation of combinatory abstraction to mark placeholders of actual names. We use $\lambda$ to mark a name as being abstract. The name that follows this is then marked in the remainder of the expression as being not a name, but rather what the name is called. We use the placeholder, until such time as we know what the name really is. Since there are initially no names at all, there will never be any names Posession.Name.* that are not $\lambda$ abstractions.

Aristotle defines a proposition as being either part of an enunciation. And he defines an enunciation as either part of a contradiction indefinitely. A contradiction is an affirmation and its corresponding denial. Then he defines the true as the propositions which state that such and such is so and so when indeed such and such *is* so and so, or which state that such and such is not so and so when indeed such and such is *not* so and so. And *vice versa* he defines the false propositions as those which state that such and such is so and so when in fact such and such is *not* so and so or which state that such and such is not so and so when in fact such and such *is* so and so. Now we have only the names of things, we do not have the things themselves in this system. Thus the system cannot decide the truth or falsity of any predication.

Therefore we treat every enunciation as being both parts of a contradiction indefinitely and so every predicate will take both parts of the enunciation and choose one as the true part and the other as the false part. Now when one predicate is defined in terms of others then the truth of these other predicates determines the truth of the predicate which we are defining. So in fact all of our predicates must refer to the truth predicate and will have to have some placeholder $p$ for the as-yet-undefined name of the truth predicate. This predicate will be a predicate like any other so must consist of both parts of a contradiction. What it needs to do is to 'conform to itself in every way' which is to say that the truth predicate applied to the truth predicate should yield the truth predicate and the truth predicate applied to the falsity predicate should yield the falsity predicate, and the falsity predicate applied to the truth predicate should yield the falsity predicate and the falsity predicate applied to the falsity predicate should yield the truth predicate. What we require is pairs $\langle p,q\rangle \equiv \mathbf{True}$ and $\langle q,p\rangle \equiv \mathbf{False}$ and a *Syllogistic* combinator **S** such that

$$\begin{array}{ll} \mathbf{S}\,\mathbf{True}\,\mathbf{True} = \mathbf{True} & \mathbf{S}\,\mathbf{False}\,\mathbf{True} = \mathbf{False} \\ \mathbf{S}\,\mathbf{True}\,\mathbf{False} = \mathbf{False} & \mathbf{S}\,\mathbf{False}\,\mathbf{False} = \mathbf{True} \end{array} \tag{3}$$

so that

$$\begin{array}{ll} \mathbf{S}\,\langle p,q\rangle\,\langle p,q\rangle = \langle p,q\rangle & \mathbf{S}\,\langle q,p\rangle\,\langle p,q\rangle = \langle q,p\rangle \\ \mathbf{S}\,\langle p,q\rangle\,\langle q,p\rangle = \langle q,p\rangle & \mathbf{S}\,\langle q,p\rangle\,\langle q,p\rangle = \langle p,q\rangle. \end{array} \tag{4}$$

This is achieved by

$$\mathbf{S} \equiv \lambda\langle p,q\rangle.\lambda x.\langle p\,x, q\,x\rangle \tag{5}$$

where we have used pattern matching in the argument $\langle p,q\rangle$ as an abbreviation for

$$\lambda x.\lambda p = \pi_1\,x.\lambda q = \pi_2\,x.\lambda x.\langle p\,x, q\,x\rangle$$

which is in turn an abbreviation for

$$\lambda x.(\lambda p.\lambda q.\lambda x.\langle p\,x, q\,x\rangle)(\pi_1\,x)(\pi_2\,x)$$

and $\pi_1$, $\pi_2$ and $\langle,\rangle$ are primitive combinators such that

$$\langle \pi_1\,x, \pi_2\,x\rangle = x.$$

If we adopt the convention that the first of the pair of a contradiction is the *affirmation* and the second the *denial* then the two kinds of predications can be represented using the abstract names

$$\mathbf{affirm} \equiv \langle \pi_1, \pi_2\rangle \qquad \text{and} \qquad \mathbf{deny} \equiv \langle \pi_2, \pi_1\rangle.$$

Then negation can be represented by the combinator

$$\mathbf{N} \equiv \lambda\langle p,q\rangle.\langle q,p\rangle \tag{6}$$

so that if $P$ is some enunciation then $\mathbf{N}P$ is its negation in the sense that whenever $P$ affirms then the negation denies and *vice versa*. All formal proofs are in terms of utterances. Demonstrations are true or false, but we can only know that a demonstration is false when the premisses are evidently true propositions and the conclusion is an evidently false proposition. This can only be judged by someone applying the formal proof to the evidently true propositons of the enunciations which are its premisses. In other words the indefinite enunciations of the premisses are instantiated by evident truth of the propositions which are the forms of those enunciations and this causes the enunciation of the conclusion to become a definite proposition whose evident truth can be judged. The evident truth of an enunciation $P$ is the enunciation $\lambda P.\mathbf{S}\,\langle \mathbf{affirm}, \mathbf{deny}\rangle\,P$ which affirms the affirmation and denies the denial of $P$. This is just the identity function so affirming '$P$ is true' is the same as as affirming $P$ itself.

The fundamental principle of logic is the principle of non-contradiction:

In any enunciation $\langle P,Q\rangle$ one of $P$ or $Q$ must be true and the other false.

We cannot prove this of course, but we can formalise it as an undefined predicate **Prop** which represents a predicate which asserts indefinitely of any pair $\langle P, Q\rangle$ that one of $P$ or $Q$ is true and the other false. So a proposition is a disjunction of predicates and the predicate **Prop** must satisfy

$$\begin{aligned}\mathbf{S}\,\mathbf{Prop}\langle\langle p,q\rangle,\langle r,s\rangle\rangle \equiv \mathbf{S}\,\mathbf{Prop}\,\langle\langle \mathbf{S}\,\mathbf{True}\,\langle p,r\rangle, \mathbf{S}\,\mathbf{False}\,\langle q,s\rangle\rangle,\\ \langle \mathbf{S}\,\mathbf{False}\,\langle p,r\rangle, \mathbf{S}\,\mathbf{True}\,\langle q,s\rangle\rangle\rangle\end{aligned} \tag{7}$$

Aristotle writes 'we say "$Q$ is predicated of all of $P$" if no instance of $P$ can be found of which $Q$ cannot be asserted, and "$Q$ is predicated of none of $P$" is to be understood in the same way'. So quantification is a conjunction of predicates.

Writing **A** and **I** for universal and existential quantification respectively, and **E** and **O** for their respective negations, quantification can be formalised in the equations

$$\begin{aligned}\mathbf{A} &\equiv \lambda\langle P,Q\rangle.\mathbf{N}(\mathbf{I}\langle P,\mathbf{N}\,Q\rangle) & \mathbf{E} &\equiv \lambda\langle P,Q\rangle.\mathbf{N}(\mathbf{I}\langle P,Q\rangle)\\ \mathbf{I} &\equiv \lambda\langle P,Q\rangle.\mathbf{N}(\mathbf{A}\langle P,\mathbf{N}\,Q\rangle) & \mathbf{O} &\equiv \lambda\langle P,Q\rangle.\mathbf{N}(\mathbf{A}\langle P,Q\rangle)\end{aligned} \tag{8}$$

Now it is easy to show that the equations (4) are satisfied if and only if

$$p = \mathbf{affirm} \qquad q = \mathbf{deny}$$

so the true and the false are

$$\begin{aligned}\mathbf{True} &\equiv \langle \mathbf{affirm}, \mathbf{deny}\rangle\\ \mathbf{False} &\equiv \langle \mathbf{deny}, \mathbf{affirm}\rangle.\end{aligned}$$

We can represent conjunction by

$$\mathbf{C} \equiv \lambda\langle\langle p,q\rangle,\langle r,s\rangle\rangle.\langle\langle p,s\rangle,\langle q,r\rangle\rangle. \tag{9}$$

The quantifier equations (8) are satisfied if

$$\mathbf{I} \equiv \lambda\langle P,Q\rangle.\langle \mathbf{S}(\mathbf{C}\,\langle P,Q\rangle)\,R, \mathbf{S}(\mathbf{Prop}\,\langle \mathbf{C}\,\langle P,\mathbf{N}\,Q\rangle, \mathbf{N}\,P\rangle)\rangle. \tag{10}$$

Note that in the affirmation there is a free variable $R$ which is a witness of which both $P$ and $Q$ can be predicated, thus $R$ is the name of a counterexample to the denial; and the term in the right hand side is partially applied in the sense that it is of the form $\lambda x.M$. This is indicative of a *universal* from which the subject $x$ has been abstracted. The $x$ is a placeholder for a counterexample to the affirmation. Note also that both this and the resulting definition of **A** are just two-place predicates like any other.

Then the quantifier conversions are

$$\begin{aligned}\mathbf{A}\langle P,Q\rangle &\Rightarrow \mathbf{I}\langle Q,P\rangle\\ \mathbf{E}\langle P,Q\rangle &\Rightarrow \mathbf{E}\langle Q,P\rangle\\ \mathbf{I}\langle P,Q\rangle &\Rightarrow \mathbf{I}\langle Q,P\rangle\end{aligned} \tag{11}$$

Now clearly this sort of trick only works when the names have an internal coherence which is the relation between the different senses in which a name is used. Scott[33] wrote in [7] 'I am trying to find out where λ-calculus *should* come from, and

[33]Quoted in [4]. This is the logician Dana Scott, not the author of *Waverley*.

the fact that the notion of a cartesian closed category is a late developing one (Eilenberg & Kelly (1966)), is not relevant to the argument: I shall try to explain in my own words in the next section why we should look to it first.' This captures exactly Aristotle's sense of the difference between the things prior and better known to man as distinct to those prior and better known in the order of being. Human knowledge proceeds from the concrete towards the abstract, but true knowledge proceeds from the abstract to the concrete. We discovered λ-calculus before we discovered modern category theory: modern category theory is more abstract. The aim of Scott and then Moggi was to show how modern category theory explains λ-calculus. Lambda calculus is the essence of abstraction and so an explanation of lambda calculus is an explanation of how we obtain abstract knowledge.

Really what we are trying to do in philosophy is to find the true names of things, which are those names that have the right 'fortuitous sense' in the other categories. If there is indeed a fixedpoint, then this will determine exactly what is the most efficient way to describe the different types of substantive objects of sense-perception. The abstract names are then exactly the abstract types of the substantive objects. These substantives don't appear in the system however because they are the actual things: the subjects of our analysis, and the other categories are the abstract products of our synthesis of particular knowledge. If we try to analyse the other categories we immediately enter an infinite regress.

But when we get it right then we have the 'infinite data compression' we are promised. So perhaps one day we will be able to say to a student 'so now you see that the reason why the right angle is the unit of Place is the same reason why One is the unit of Quantity. And this is one step—one transition across a boundary—in the student's realization of the one whole Platonic Form whose name is called 'unit'. Then what the student has is one unified explanation of the fundamental elements of geometry and of arithmetic: that some of the demonstrations of geometrical knowledge have a one to one correspondence with others of arithmetic. These then are two different names for the same actual thing. The actual thing is the Form whose name is 'Unit', and the different names are the sense that the word 'unit' is used in geometry and the sense that the word 'unit' is used in arithmetic. It is just one word, and it has one definite meaning in each context in which it is used. It is this context that does the work of deciding what meaning the word has, and it is always we who provide the context.

Thus an ultimate explanation for the elements of arithmetic being what they are will be the same as the ultimate explanation for the elements of geometry being what they are. Thus the structure of the explanations of our sense-perception is determined by something very abstract which is what we call *logic*, which is just the Greek word for speech or discourse. It the laws of discourse which ultimately determine the structure of scientific knowledge because scientific knowledge is based on the foundation of descriptions of sense-perception and to be effective these descriptions must be definite. The ultimate law of logic is the law of non-contradiction: either something is so or it is not so. There cannot be anything in-between because we require our knowledge to have a definite foundation in statements of fact that can be verified in an act of evident judgement by any one who understands these statements.

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